Decomposing MWR - an Update

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Introduction

Initial comments on MWR

Money-weighted rate of return (MWR) measures the return of a portfolio in a way that the return is sensitive to changes in the money invested:

- MWR measures the return from a client's perspective where he does have control over the (external) cash flows.
- MWR does not allow a comparison across peer groups.
- MWR does allow a comparison against a benchmark (adjusted for cash flows).
- MWR is best measured by the internal rate of return (IRR).
- calculating, decomposing and reporting MWRs is not common practice.
- MWRs are not generally covered by the GIPS Standards - just for private equity and closed end real estate funds.

Decomposition of MWR versus TWR

The MWR allows a decomposition of the portfolio return reflecting the client's main investment decision:

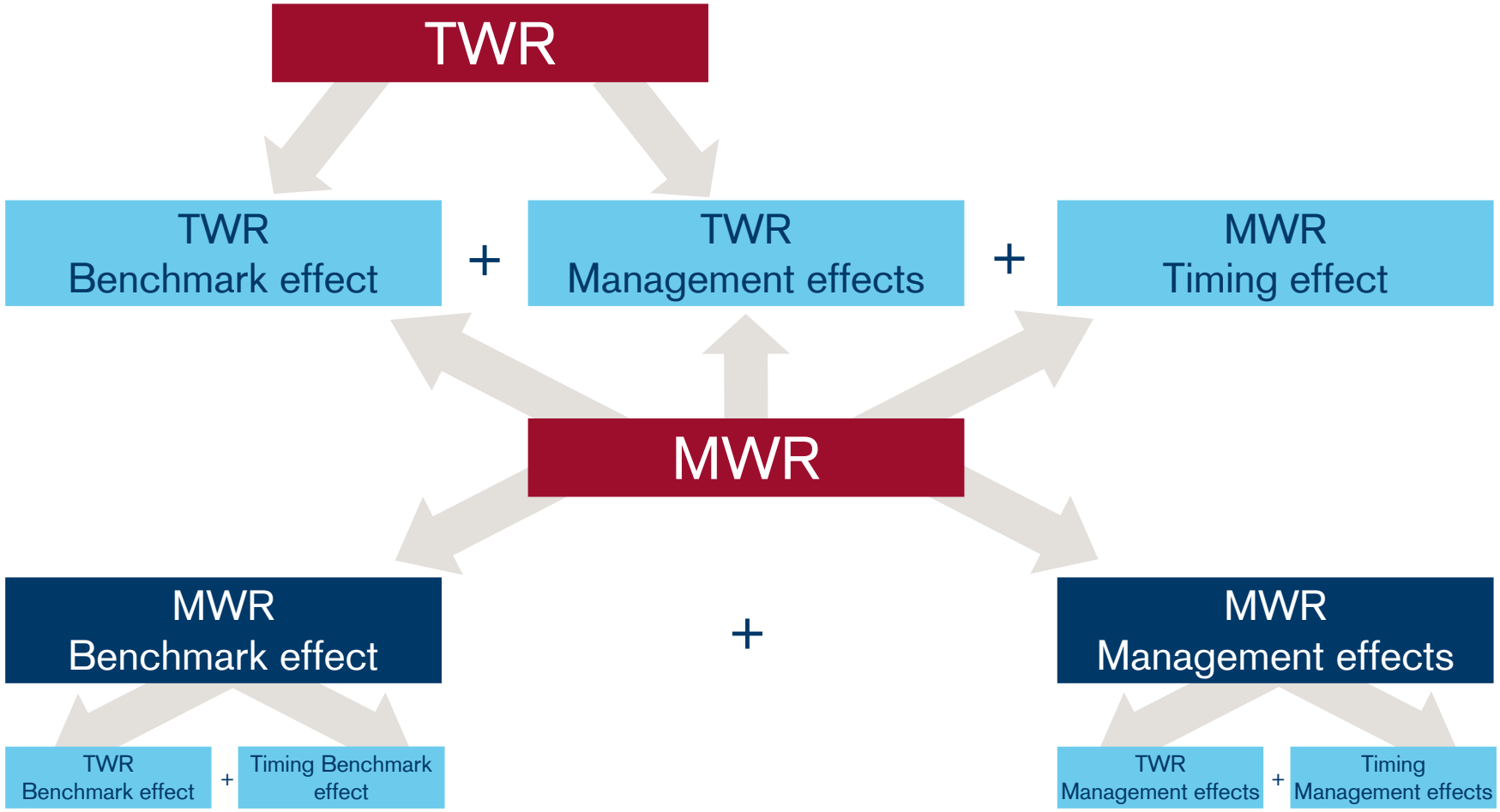
- TWR Benchmark effect => reflects the return contribution based on the client's decision to invest his initial capital into a specific benchmark strategy (corresponds to the TWR benchmark return).
- TWR Management effect => reflects the return contribution based on deviating from the benchmark strategy by asset allocation and stock picking (corresponds to the TWR attribution effects).
- MWR Timing effect => reflects the return contribution of changing the initial invested capital into the benchmark strategy and into the asset allocation of the portfolio (corresponds to the difference between MWR and TWR).

Decomposition of MWR - another perspective

The MWR allows also a decomposition of the portfolio return without explicitly separating the timing effect:

- MWR Benchmark effect => reflects the return contribution based on the client's decision to invest his capital into a specific benchmark strategy (including the effect of changing the initial invested capital).
- MWR Management effect => reflects the return contribution based on deviating from the benchmark strategy by asset allocation and stock picking (including the effect of changing the initial invested capital).

Comparison of different decomposition approaches



Calculation of MWR

First step towards MWR-Attribution

- Calculation of the MWR for the portfolio.
- Calculation of the MWR for the benchmark

  => by simulating the portfolio's cash in- and outflows also for the benchmark.
- Calculation of the excess MWR.

Calculation of the MWR for the portfolio

$$\frac{EMV_P}{(1+IRR_P)^T} + \left(\sum_{t=1}^{T-1} \frac{-C_{P,t}}{(1+IRR_P)^t} \right) - BMV_P = 0$$

with

IRR_P	=	IRR of portfolio,
BMV_P	=	Portfolio beginning market value,
EMV_P	=	Portfolio ending market value at T,
$C_{P,t}$	=	Portfolio cash flow at t.

To calculate the MWR, in the industry different methodologies are used where all but one are approximation methods for the “true” MWR. In the following the internal rate of return methodology (IRR) as the "true" MWR is used because it is not only the most precise method for calculating a MWR but the one methodology that solves the full calculation problem. The IRR is the return / interest rate that causes the ending market value and intermediate cash flows to be discounted to the beginning market value.

Calculation of the MWR for the benchmark

$$\frac{EMV_B}{(1+IRR_B)^T} + \left(\sum_{t=1}^{T-1} \frac{-C_{B,t}}{(1+IRR_B)^t} \right) - BMV_B = 0$$

with

IRR_B	=	IRR of benchmark,
BMV_B	=	Benchmark beginning market value,
EMV_B	=	Benchmark ending market value at T,
$C_{B,t}$	=	Benchmark cash flow at t.

Here it is important that the cash inflows (outflows) are invested (de-invested) according to the actual benchmark asset allocation at the time of the cash flow and that the returns of the money invested equal the respective returns of the underlying benchmark investments. In addition cash flows have to be simulated for rebalancing activities.

Calculation of the MWR excess return

$$EIRR_P = IRR_P - IRR_B$$

with

$EIRR_P$ = Excess IRR.

Contribution to MWR

Second step towards MWR-Attribution

- Calculation of the profit and loss of the different asset classes.
- Calculation of the average invested capital for the different asset classes.
- Calculation of the asset class contribution to the MWR for the portfolio.
- Calculation of the asset class contribution to the MWR for the benchmark.
- Calculation of the asset class contribution to the excess MWR.

Calculation of the profit and loss

$$PL_P = EMV_P - BMV_P - \sum_{t=1}^{T-1} C_{P,t}$$

with

PL_P = P & L of portfolio.

$$PL_P = \sum_{i=1}^{n} PL_{P,i} = \sum_{i=1}^{n} \left(EMV_{P,i} - BMV_{P,i} - \sum_{t=1}^{T-1} C_{P,i,t} \right)$$

with

$PL_{P,i}$ = P & L of asset class i,

$EMV_{P,i}$ = Ending market value of asset class i,

$BMV_{P,i}$ = Beginning market value of asset class i,

$C_{P,i,t}$ = Cash flow of asset class i at t,

n = Number of asset classes.

Same formulas apply for the benchmark

Calculation of the average invested capital

$$AIC_P = \frac{PL_P}{IRR_P}$$

with

AIC_P = Average invested capital of portfolio.

$$AIC_{P,i} = \frac{PL_{P,i}}{IRR_{P,i}}$$

with

$AIC_{P,i}$ = Average invested capital of asset class i,

$IRR_{P,i}$ = IRR of asset class i.

It is important to note that the average invested capital of the total portfolio does not have to be equal to the sum of the average invested capitals of all asset classes.

$$AIC_P \leq \text{or} = \text{or} \geq \sum_{i=1}^{n} AIC_{P,i}$$

Same formulas apply for the benchmark

Calculation of the contribution to the MWR for the portfolio

$$
\begin{aligned}
IRR_P &= \frac{PL_P}{AIC_P} \\
&= \sum_{i=1}^{n} \frac{PL_{P,i}}{AIC_P} \\
&= \sum_{i=1}^{n} \frac{AIC_{P,i}}{AIC_P} \times IRR_{P,i} \\
&= \sum_{i=1}^{n} RC_{P,i}
\end{aligned}
$$

with

$RC_{P,i}$ = Return contribution of asset class i.

Within the IRR framework every cash flow series can be transferred to a cash flow series consisting of two cash flows - the cash inflow at the beginning of the investment period and a cash outflow at the end of the investment period. For such a cash flow series the average invested capital is equal to the cash inflow at the beginning of the investment period.

Calculation of the contribution to the MWR for the benchmark

$$
\begin{aligned}
IRR_B &= \frac{PL_B}{AIC_B} \\
&= \sum_{i=1}^{n} \frac{PL_{B,i}}{AIC_B} \\
&= \sum_{i=1}^{n} \frac{AIC_{B,i}}{AIC_B} \times IRR_{B,i} \\
&= \sum_{i=1}^{n} RC_{B,i}
\end{aligned}
$$

with

$RC_{B,i}$	=	Return contribution of asset class i,
PL_B	=	P & L of benchmark,
$PL_{B,i}$	=	P & L of asset class i,
AIC_B	=	Average invested capital of benchmark,
$AIC_{B,i}$	=	Average invested capital of asset class i.

Calculation of the contribution to the excess MWR

$$
\begin{aligned}
EIRR_P &= IRR_P - IRR_B \\
&= \sum_{i=1}^{n} RC_{P,i} - \sum_{i=1}^{n} RC_{B,i} \\
&= \sum_{i=1}^{n} \frac{AIC_{P,i}}{AIC_P} \times IRR_{P,i} - \sum_{i=1}^{n} \frac{AIC_{B,i}}{AIC_B} \times IRR_{B,i} \\
&= \sum_{i=1}^{n} \frac{PL_{P,i}}{AIC_P} - \sum_{i=1}^{n} \frac{PL_{B,i}}{AIC_B}
\end{aligned}
$$

MWR-Attribution

Last step towards MWR-Attribution

Here the excess MWR is decomposed according to the Brinson, Hood and Beebower return attribution methodology and therefore split up into the asset allocation effect, stock picking effect and interaction effect.

$$
\begin{aligned}
EIRR_P &= IRR_P - IRR_B \\
&= AAE_P + SPE_P + IAE_P \\
&= \sum_{i=1}^{n} AAE_{P,i} + \sum_{i=1}^{n} SPE_{P,i} + \sum_{i=1}^{n} IAE_{P,i}
\end{aligned}
$$

with

AAE_P	=	Asset allocation effect of portfolio,
SPE_P	=	Stock picking effect of portfolio,
IAE_P	=	Interaction effect of portfolio,
$AAE_{P,i}$	=	Asset allocation effect of asset class i,
$SPE_{P,i}$	=	Stock picking effect of asset class i,
$IAE_{P,i}$	=	Interaction effect of asset class i.

Simple framework for MWR-Attribution (1/4)

		Selection	
		Actual	Passive
Asset Allocation	Actual	**Quadrant IV** IRR of actual portfolio	**Quadrant II** IRR of notional portfolio 1 => active asset allocation portfolio
	Passive	**Quadrant III** IRR of notional portfolio 2 => active stock picking portfolio	**Quadrant I** IRR of benchmark

Simple framework for MWR-Attribution (2/4)

Quadrant IV represents the IRR of the portfolio where for the calculation all (de-) investments made in each asset class – based on the actual weights of the asset class and expressed as cash flows - and the actual returns for each asset class are needed.	Quadrant II represents the IRR of the notional portfolio 1 which reflects the active asset allocation of the portfolio assuming no stock picking. To calculate the IRR of the notional portfolio 1 all (de-) investments made in each asset class – based on the actual weights of the asset class and expressed as cash flows - and the passive index returns for each asset class are needed.
Quadrant III represents the IRR of the notional portfolio 2 which reflects the active stock picking of the portfolio assuming no active asset allocation. To calculate the IRR of the notional portfolio 2 all (de-) investments made in each asset class – based on the passive weights of the asset class and expressed as cash flows - and the actual returns for each asset class are needed.	Quadrant I represents the IRR of the benchmark where for the calculation all (de-) investments made in each asset class – based on the passive weights of the asset class and expressed as cash flows - and the passive index returns for each asset class are needed.

Simple framework for MWR-Attribution (3/4)

$$AAE_P = \text{Quadrant II} - \text{Quadrant I}$$
$$= IRR_{NP1} - IRR_B$$
$$= \sum_{i=1}^{n} RC_{NP1,i} - \sum_{i=1}^{n} RC_{B,i}$$

with

IRR_{NP1} = IRR of notional portfolio 1,

$RC_{NP1,i}$ = Return contribution of asset class i.

$$SPE_P = \text{Quadrant III} - \text{Quadrant I}$$
$$= IRR_{NP2} - IRR_B$$
$$= \sum_{i=1}^{n} RC_{NP2,i} - \sum_{i=1}^{n} RC_{B,i}$$

with

IRR_{NP2} = IRR of notional portfolio 2,

$RC_{NP2,i}$ = Return contribution of asset class i.

Simple framework for MWR-Attribution (4/4)

$$
\begin{aligned}
IAE_P &= \text{Quadrant IV} - \text{Quadrant III} \\
&\quad - \text{Quadrant II} + \text{Quadrant I} \\
&= IRR_P - IRR_{NP2} \\
&\quad - IRR_{NP1} + IRR_B \\
&= \sum_{i=1}^{n} RC_{P,i} - \sum_{i=1}^{n} RC_{NP2,i} \\
&\quad - \sum_{i=1}^{n} RC_{NP1,i} + \sum_{i=1}^{n} RC_{B,i}
\end{aligned}
$$

On an asset class level:

$$AAE_{P,i} = RC_{NP1,i} - RC_{B,i}$$

$$SPE_{P,i} = RC_{NP2,i} - RC_{B,i}$$

$$IAE_{P,i} = RC_{P,i} - RC_{NP2,i} - RC_{NP1,i} + RC_{B,i}$$

Profit and Loss Attribution

Last step towards Profit and Loss Attribution

Here the excess profit and loss is decomposed according to the Brinson, Hood and Beebower return attribution methodology and therefore split up into the asset allocation effect, stock picking effect and interaction effect.

$$
\begin{aligned}
EPL_P &= PL_P - PL_B \\
&= AAPL_P + SPPL_P + IAPL_P \\
&= \sum_{i=1}^{n} AAPL_{P,i} + \sum_{i=1}^{n} SPPL_{P,i} + \sum_{i=1}^{n} IAPL_{P,i}
\end{aligned}
$$

with

$AAPL_P$ = P & L due to asset allocation of portfolio,

$SPPL_P$ = P & L due to stock picking of portfolio,

$IAPL_P$ = P & L due to interaction effect of portfolio,

$AAPL_{P,i}$ = P & L due to asset allocation of asset class i,

$SPPL_{P,i}$ = P & L due to stock picking of asset class i,

$IAPL_{P,i}$ = P & L due to interaction of asset class i.

Simple framework for Profit and Loss Attribution (1/4)

		Selection	
		Actual	Passive
Asset Allocation	Actual	**Quadrant IV** P&L of actual portfolio	**Quadrant II** P&L of notional portfolio 1 => active asset allocation portfolio
	Passive	**Quadrant III** P&L of notional portfolio 2 => active stock picking portfolio	**Quadrant I** P&L of benchmark

Simple framework for Profit and Loss Attribution (2/4)

Quadrant IV represents the P&L of the portfolio where for the calculation all (de-) investments made in each asset class – based on the actual weights of the asset class and expressed as cash flows - and the actual returns for each asset class are needed.	Quadrant II represents the P&L of the notional portfolio 1 which reflects the active asset allocation of the portfolio assuming no stock picking. To calculate the P&L of the notional portfolio 1 all (de-)investments made in each asset class – based on the actual weights of the asset class and expressed as cash flows - and the passive index returns for each asset class are needed.
Quadrant III represents the P&L of the notional portfolio 2 which reflects the active stock picking of the portfolio assuming no active asset allocation. To calculate the P&L of the notional portfolio 2 all (de-)investments made in each asset class – based on the passive weights of the asset class and expressed as cash flows - and the actual returns for each asset class are needed.	Quadrant I represents the P&L of the benchmark where for the calculation all (de-) investments made in each asset class – based on the passive weights of the asset class and expressed as cash flows - and the passive index returns for each asset class are needed.

Simple framework for Profit and Loss Attribution (3/4)

$$AAPL_P = PL_{NP1} - PL_B = \sum_{i=1}^{n} PL_{NP1,i} - \sum_{i=1}^{n} PL_{B,i}$$

with

PL_{NP1} = P & L of notional portfolio 1,

$PL_{NP1,i}$ = P & L of asset class i.

$$SPPL_P = PL_{NP2} - PL_B = \sum_{i=1}^{n} PL_{NP2,i} - \sum_{i=1}^{n} PL_{B,i}$$

with

PL_{NP2} = P & L of notional portfolio 2,

$PL_{NP2,i}$ = P & L of asset class i.

Simple framework for Profit and Loss Attribution (4/4)

$$
\begin{aligned}
IAPL_P &= PL_P - PL_{NP2} \\
&\quad - PL_{NP1} + PL_B \\
&= \sum_{i=1}^{n} PL_{P,i} - \sum_{i=1}^{n} PL_{NP2,i} \\
&\quad - \sum_{i=1}^{n} PL_{NP1,i} + \sum_{i=1}^{n} PL_{B,i}
\end{aligned}
$$

On an asset class level:

$$AAPL_{P,i} = PL_{NP1,i} - PL_{B,i}$$

$$SPPL_{P,i} = PL_{NP2,i} - PL_{B,i}$$

$$IAE_{P,i} = PL_{P,i} - PL_{NP2,i} - PL_{NP1,i} + PL_{B,i}$$

Hypothetical Example

Assumptions

- Sample multi-asset class portfolio is investing in two asset classes A and B.
- Relevant benchmark is also investing in these two asset classes A and B.
- The portfolio as well the benchmark are rebalanced on a yearly basis at the beginning of the calendar year.
- A two year period from 31.12.2006 until 31.12.2008 is considered.
- At the beginning of 2007 EUR 150 are invested in the portfolio.
- At the beginning of 2008 additional EUR 100 are invested into the portfolio according to the then current active asset allocation and stock pickings.

Return calculations (1/2)

Actual Portfolio (IRR)			
	Period 1	**Period 2**	
Dates	31.12.2006	31.12.2007	31.12.2008
	Cash flow at beginning of period	Cash flow at beginning of period	Market value at the end of period
Asset A	-75.0	47.6	36.7
Asset B	-75.0	-147.6	240.8
Portfolio	-150.0	-100.0	277.5
	Actual weights at beginning of period	Actual weights at beginning of period	Weights at the end of period
Asset A	**50.0%**	**15.0%**	13.2%
Asset B	**50.0%**	**85.0%**	86.8%
Portfolio	100.0%	100.0%	100.0%
	Actual return	Actual return	Cummulative return
Asset A	**15.0%**	**-5.0%**	17.9%
Asset B	**-5.0%**	**10.0%**	12.4%
Portfolio	5.0%	7.8%	13.8%

Benchmark (IRR)			
	Period 1	**Period 2**	
Dates	31.12.2006	31.12.2007	31.12.2008
	Cash flow at beginning of period	Cash flow at beginning of period	Market value at the end of period
Asset A	-45.0	-39.5	83.0
Asset B	-105.0	-60.6	167.2
Portfolio	-150.0	-100.0	250.2
	Passive weights at beginning of period	Passive weights at beginning of period	Weights at the end of period
Asset A	**30.0%**	**30.0%**	33.2%
Asset B	**70.0%**	**70.0%**	66.8%
Portfolio	100.0%	100.0%	100.0%
	Passive return	Passive return	Cummulative return
Asset A	**-20.0%**	**10.0%**	-2.2%
Asset B	**10.0%**	**-5.0%**	1.3%
Portfolio	1.0%	-0.5%	0.1%

Remark: negative (positive) cash flow means cash inflow (outflow).

Return calculations (2/2)

Notional Portfolio 1 (IRR)			
	Period 1	**Period 2**	
Dates	31.12.2006	31.12.2007	31.12.2008
	Cash flow at beginning of period	Cash flow at beginning of period	Market value at the end of period
Asset A	-75.0	23.6	40.0
Asset B	-75.0	-123.6	195.8
Portfolio	-150.0	-100.0	235.8
	Actual weights at beginning of period	Actual weights at beginning of period	Weights at the end of period
Asset A	**50.0%**	**15.0%**	17.0%
Asset B	**50.0%**	**85.0%**	83.0%
Portfolio	100.0%	100.0%	100.0%
	Passive return	Passive return	Cummulative return
Asset A	**-20.0%**	**10.0%**	-18.2%
Asset B	**10.0%**	**-5.0%**	-2.0%
Portfolio	-5.0%	-2.8%	-7.0%

Notional Portfolio 2 (IRR)			
	Period 1	**Period 2**	
Dates	31.12.2006	31.12.2007	31.12.2008
	Cash flow at beginning of period	Cash flow at beginning of period	Investment at the end of period
Asset A	-45.0	-23.7	71.7
Asset B	-105.0	-76.3	193.7
Portfolio	-150.0	-100.0	265.3
	Passive weights at beginning of period	Passive weights at beginning of period	Weights at the end of period
Asset A	**30.0%**	**30.0%**	27.0%
Asset B	**70.0%**	**70.0%**	73.0%
Portfolio	100.0%	100.0%	100.0%
	Actual return	Actual return	Cummulative return
Asset A	**15.0%**	**-5.0%**	5.2%
Asset B	**-5.0%**	**10.0%**	8.7%
Portfolio	1.0%	5.5%	7.7%

Remark: negative (positive) cash flow means cash inflow (outflow).

Management effects

Actual Portfolio (IRR)				
	P&L	AIC	IRR	RC
Asset A	9.3	52.1	17.9%	4.7%
Asset B	18.1	146.8	12.4%	9.1%
Portfolio	27.5	198.4	13.8%	13.8%

Notional Portfolio 1 (IRR)				
	P&L	AIC	IRR	RC
Asset A	-11.4	62.6	-18.2%	-5.7%
Asset B	-2.8	137.2	-2.0%	-1.4%
Portfolio	-14.2	201.0	-7.0%	-7.0%

Notional Portfolio 2 (IRR)				
	P&L	AIC	IRR	RC
Asset A	3.0	56.7	5.2%	1.5%
Asset B	12.4	142.4	8.7%	6.2%
Portfolio	15.3	199.1	7.7%	7.7%

Benchmark (IRR)				
	P&L	AIC	IRR	RC
Asset A	-1.5	64.9	-2.2%	-0.7%
Asset B	1.7	135.2	1.3%	0.8%
Portfolio	0.2	200.1	0.1%	0.1%

IRR-Attribution				
	AAE	SPE	IAE	Total
Asset A	-4.9%	2.2%	8.1%	5.4%
Asset B	-2.2%	5.4%	5.2%	8.3%
Excess	-7.2%	7.6%	13.3%	13.7%

Profit and Loss Attribution				
	AAPL	SPPL	IAPL	Total
Asset A	-9.9	4.4	16.2	10.8
Asset B	-4.5	10.7	10.3	16.4
Excess	-14.4	15.1	26.5	27.2

Simple Example for an IRR Implementation

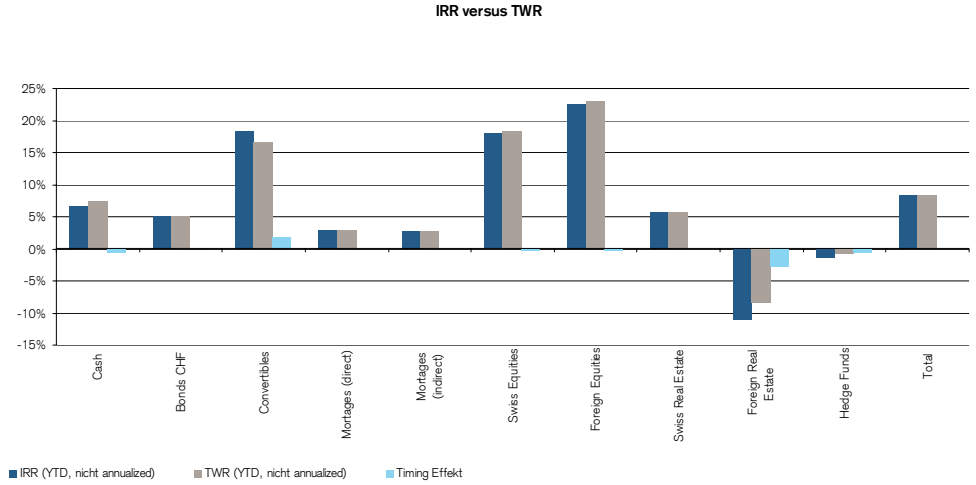
Simple example for an IRR implementation (1/2)

Asset Class	Asset at 31.12.08	Sum of cash flows	Assets at 30.12.2009	Profit & loss (in CHF)	Profit & loss (in %)	Sum of cash flows in % of assets at 31.12.2008	On average invested capital (in CHF)	On average invested capital (in %)
Cash	77'299'536	-44'553'165	37'028'584	4'282'214	6.77%	-57.64%	64'195'773	8.50%
Bonds CHF	90'532'962	-14'375'791	80'742'922	4'585'751	7.25%	-15.88%	89'041'307	11.79%
Convertibles	11'989'017	12'524'530	28'169'285	3'655'738	5.78%	104.47%	19'773'155	2.62%
Mortages (direct)	88'682'597	-2'828'500	88'446'889	2'592'792	4.10%	-3.19%	86'973'765	11.52%
Mortages (indirect)	78'401'686	-11'744'126	68'594'225	1'936'665	3.06%	-14.98%	71'637'225	9.49%
Swiss Equities	88'308'678	15'142'362	119'931'920	16'480'880	26.04%	17.15%	91'359'125	12.10%
Foreign Equities	82'870'803	37'716'722	141'375'901	20'788'376	32.85%	45.51%	92'020'100	12.19%
Swiss Real Estate	204'598'112	-26'089'690	189'673'326	11'164'904	17.64%	-12.75%	193'225'914	25.59%
Foreign Real Estate	19'409'158	-4'485'424	13'162'380	-1'761'354	-2.78%	-23.11%	15'751'996	2.09%
Hedge Funds	17'680'187	35'756'596	52'996'707	-440'076	-0.70%	202.24%	30'541'714	4.05%
Total	**759'772'736**	**-2'936'486**	**820'122'139**	**63'285'889**	**100.00%**	**-0.39%**	**755'014'682**	**100.00%**

Data needed:
- TWRs and TWR contributions
- Month end valuations
- Cash flows during the month

Simple example for an IRR implementation (2/2)

Asset Class	Profit & loss (in CHF)	IRR (YTD, nicht annualized)	IRR Contribution	Timing Effekt	TWR (YTD, nicht annualized)
Cash	4'282'214	6.67%	0.57%	-0.69%	7.36%
Bonds CHF	4'585'751	5.15%	0.61%	-0.01%	5.16%
Convertibles	3'655'738	18.49%	0.48%	1.89%	16.60%
Mortages (direct)	2'592'792	2.98%	0.34%	0.01%	2.97%
Mortages (indirect)	1'936'665	2.70%	0.26%	0.00%	2.70%
Swiss Equities	16'480'880	18.04%	2.18%	-0.44%	18.48%
Foreign Equities	20'788'376	22.59%	2.75%	-0.44%	23.03%
Swiss Real Estate	11'164'904	5.78%			
Foreign Real Estate	-1'761'354	-11.18%			
Hedge Funds	-440'076	-1.44%			
Total	**63'285'889**	**8.38%**			



Comments and Questions

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