

# **Investigating the Case for MWR-Attribution**

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# Agenda

(1/2)

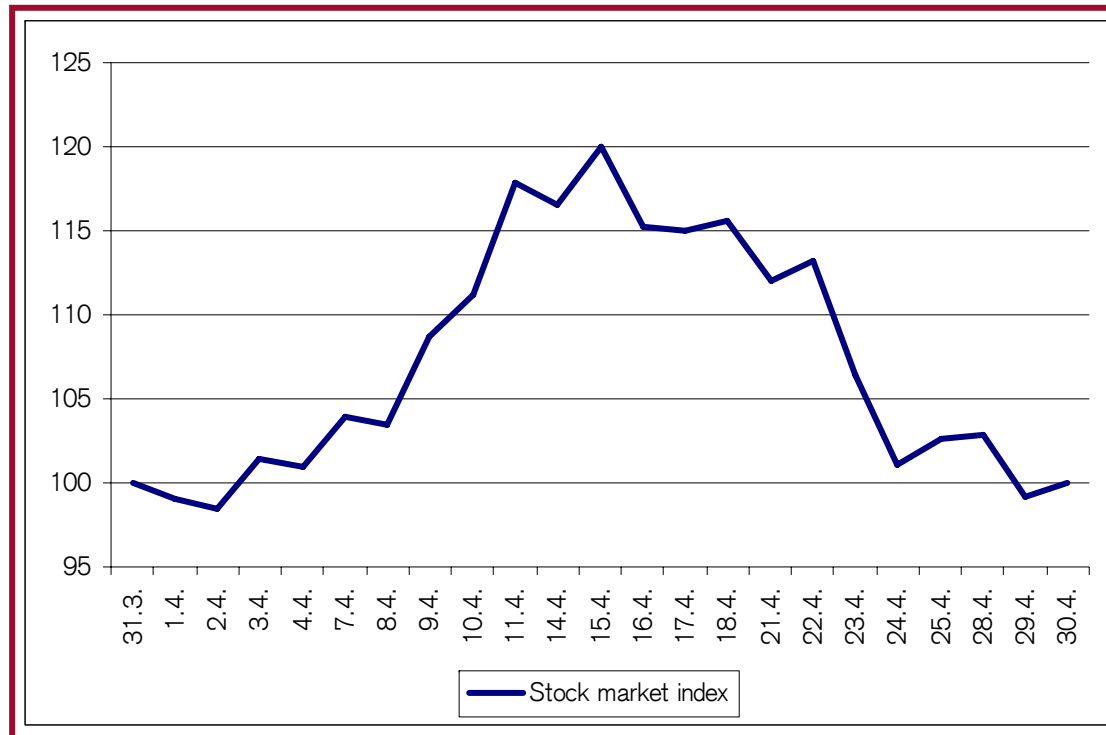
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# Agenda

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# Comparison of TWR versus MWR



What is the return of the stock market index or of an index portfolio manager?

**=> TWR => 0%**

What is the return of a client who doubled the money invested in the middle of the period?

**=> MWR => -11%**

## Initial comments on TWR

Time-weighted rate of return (TWR) measures the return of a portfolio in a way that the return is **insensitive** to changes in the money invested:

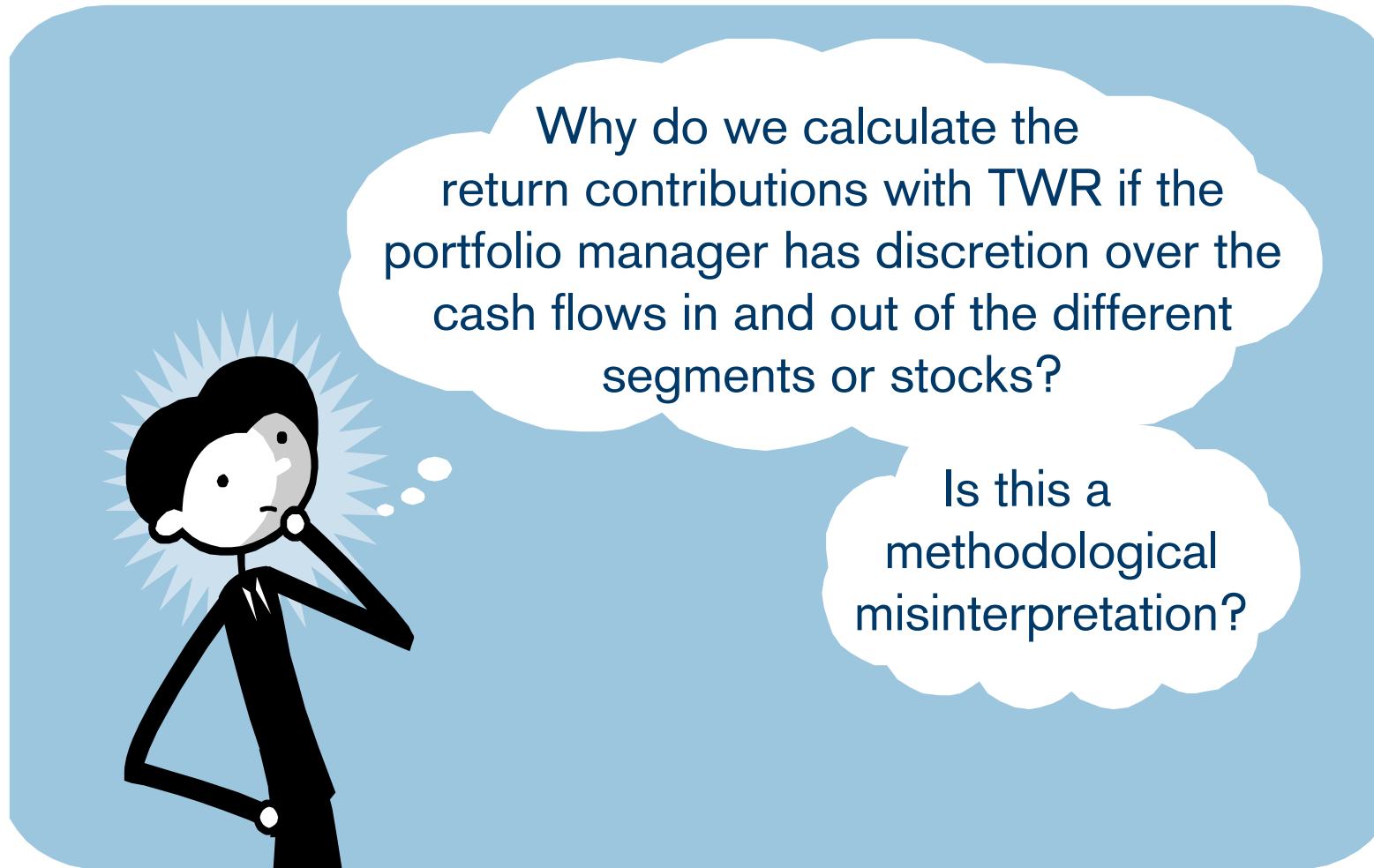
- TWR measures the return from a portfolio manager's perspective if he does not have control over the (external) cash flows.
- TWR allows a comparison against a benchmark and across peer groups.
- calculating, decomposing and reporting TWRs is common practice.
- presenting TWRs is one of the key principles of the GIPS Standards.

## Initial comments on MWR

Money-weighted rate of return (MWR) measures the return of a portfolio in a way that the return is **sensitive** to changes in the money invested:

- MWR measures the return from a client's perspective where he does have control over the (external) cash flows.
- MWR does not allow a comparison across peer groups.
- MWR does allow a comparison against a benchmark (adjusted for cash flows).
- MWR is best measured by the internal rate of return (IRR).
- calculating, decomposing and reporting MWRs is not common practice.
- MWRs are not covered by the GIPS Standards.

## Case for action -1-



## Initial comments on TWR-attribution

Decomposing the TWR is common practice and the main method implemented by performance attribution software providers, means that:

- Portfolio and benchmark returns are TWRs.
- Segment and stock returns are TWRs.
- Return contributions are calculated using TWRs.
- Use of TWRs assumes that portfolio manager has no discretion over any (external as well as internal) cash flows.
- Impact of internal as well as external cash flows are neutralized.
- Impact of over- / underweighting of segments or stocks is dealt by using weights instead of cash flows.



## Initial comments on MWR-attribution

Decomposing the MWR is not common practice and not offered by performance attribution software providers, means that:

- Decomposing the MWR or TWR using the "MWR-concept" is not common practice.
- The effect of cash flows is not allocated properly.
- The management effects may be misleading.

# TWR-attribution - some formulas

(1/3)  
for a single period

$$(1) ER_{\text{Total}} = R_p - R_b$$

$$(2) ER_{\text{Total}} = AAE_{\text{Total}} + SPE_{\text{Total}} + IAE_{\text{Total}}$$

$$(3) ER_i = AAE_i + SPE_i + IAE_i$$

$$(4) AAE_{\text{Total}} = \sum_{i=1}^n AAE_i = \sum_{i=1}^n (w_p^i - w_b^i) * r_b^i = \sum_{i=1}^n [RC_i(w_p^i; r_b^i) - RC_i(w_b^i; r_b^i)]$$

$$(5) SPE_{\text{Total}} = \sum_{i=1}^n SPE_i = \sum_{i=1}^n (r_p^i - r_b^i) * w_b^i = \sum_{i=1}^n [RC_i(w_b^i; r_p^i) - RC_i(w_b^i; r_b^i)]$$

$$(6) IAE_{\text{Total}} = \sum_{i=1}^n IAE_i = \sum_{i=1}^n [(w_p^i - w_b^i) * (r_p^i - r_b^i)]$$
$$= \sum_{i=1}^n [(w_p^i * r_p^i - w_b^i * r_p^i) - (w_p^i * r_b^i - w_b^i * r_b^i)]$$
$$= \sum_{i=1}^n \left\{ [RC_i(w_p^i; r_p^i) - RC_i(w_b^i; r_p^i)] - [RC_i(w_p^i; r_b^i) - RC_i(w_b^i; r_b^i)] \right\}$$

=> Reference for the attribution approach: "Determinants of portfolio performance" by G.P. Brinson, L.R. Hood and G.L. Beebower (1986)

## TWR-attribution - some formulas

(2/3)  
for multi-period

$$(7) \text{ERM}_{\text{Total}}^T = \text{ERM}_{\text{Total}}^{T-1} * (1 + R_b^T) + \text{ER}_{\text{Total}}^T * (1 + \text{RM}_p^{T-1})$$

$$(8) \text{ERM}_{\text{Total}}^T = \text{AAEM}_{\text{Total}}^T + \text{SPEM}_{\text{Total}}^T + \text{IAEM}_{\text{Total}}^T$$

$$(9) \text{AAEM}_{\text{Total}}^T = \text{AAEM}_{\text{Total}}^{T-1} * (1 + R_b^T) + \text{AAE}_{\text{Total}}^T * (1 + \text{RM}_p^{T-1})$$

$$(10) \text{SPEM}_{\text{Total}}^T = \text{SPEM}_{\text{Total}}^{T-1} * (1 + R_b^T) + \text{SPE}_{\text{Total}}^T * (1 + \text{RM}_p^{T-1})$$

$$(11) \text{IAEM}_{\text{Total}}^T = \text{IAEM}_{\text{Total}}^{T-1} * (1 + R_b^T) + \text{IAE}_{\text{Total}}^T * (1 + \text{RM}_p^{T-1})$$

Multi-period contribution =

[prior cumulated contribution x (1 + benchmark return current period)]

+ [current period contribution x (1 + prior cumulated portfolio return)]

=> Reference for the linking approach: "Investment Performance Measurement"; by Bruce J. Feibel

# TWR-attribution - some formulas

(3/3)  
notations

$i$  = asset class  $i$

$p; b$  = portfolio; benchmark

$R_p$  = portfolio return

$R_b$  = benchmark return

$w_{p;b}^i$  = weight asset class  $i$

$r_{p;b}^i$  = return of asset class  $i$

$ER_{i;Total}$  = excess return

$AAE_{i;Total}$  = asset allocation effect

$SPE_{i;Total}$  = stock picking effect

$IAE_{i;Total}$  = interaction effect

$RC_i$  = return contribution

$T$  = end date of the multi period

$RM_p$  = cumulated portfolio return

$ERM_{i;Total}$  = cumulated excess return

$AAEM_{i;Total}$  = cumulated asset allocation effect

$SPE_{i;Total}$  = cumulated stock picking effect

$IAE_{i;Total}$  = cumulated interaction effect

# TWR-attribution - example without external cash flows (1/6)

Portfolio			
	Period 1	Period 2	
Dates	31.12.2007	31.12.2008	31.12.2009
	Investment at beginning of period	Investment at beginning of period	Investment at the end of period
Asset A	135.00	18.75	15.00
Asset B	15.00	168.75	185.63
Total assets	150.00	187.50	200.63
	weights at beginning of period	weights at beginning of period	weights at the end of period
Asset A	<b>90.00%</b>	<b>10.00%</b>	7.48%
Asset B	<b>10.00%</b>	<b>90.00%</b>	92.52%
Total assets	100.00%	100.00%	100.00%
	Returns for period	Returns for period	Cummulative Returns (TWR)
Asset A	<b>30.00%</b>	<b>-20.00%</b>	4.00%
Asset B	<b>-20.00%</b>	<b>10.00%</b>	-12.00%
Total assets	25.00%	7.00%	33.75%

no external cash flows

# TWR-attribution - example without external cash flows (2/6)

Benchmark			
	Period 1	Period 2	
Dates	31.12.2007	31.12.2008	31.12.2009
	Investment at beginning of period	Investment at beginning of period	Investment at the end of period
Asset A	15.00	18.60	24.18
Asset B	135.00	167.40	117.18
Total assets	150.00	186.00	141.36
	weights at beginning of period	weights at beginning of period	weights at the end of period
Asset A	<b>10.00%</b>	<b>10.00%</b>	17.11%
Asset B	<b>90.00%</b>	<b>90.00%</b>	82.89%
Total assets	100.00%	100.00%	100.00%
	Returns for period	Returns for period	Cummulative Returns (TWR)
Asset A	<b>-30.00%</b>	<b>30.00%</b>	-9.00%
Asset B	<b>30.00%</b>	<b>-30.00%</b>	-9.00%
Total assets	24.00%	-24.00%	-5.76%

no external cash flows

## TWR-attribution - example without external cash flows (3/6)

	Asset A	Asset B	Total effect
Asset allocation effect	-18.24%	-18.24%	-36.48%
Stock picking effect	-1.69%	10.80%	9.11%
Interaction effect	36.48%	30.40%	66.88%
<i>Total excess return</i>	16.55%	22.96%	39.51%

Why do we have a negative stock picking effect for asset A?

$$\text{SPE (A, 1)} \Rightarrow (30\% - (-30\%)) * 10\% = 6\%$$

$$\text{SPE (A, 2)} \Rightarrow ((-20\%) - 30\%) * 10\% = -5\%$$

$$\text{SPE (A, 1+2)} \Rightarrow 6\% * (1 + (-24\%)) + (-5\%) * (1 + 25\%)$$

$$\Rightarrow 6\% - 1.44\% - 5\% - 1.25\% = -1.69\%$$

=> It seems that the stock picking for asset A added no value despite the fact that we had invested 90% of the assets in asset A in period 1!

## TWR-attribution - example without external cash flows (4/6)

	Asset A	Asset B	Total effect
Asset allocation effect	-18.24%	-18.24%	-36.48%
Stock picking effect	-1.69%	10.80%	9.11%
Interaction effect	36.48%	30.40%	66.88%
<i>Total excess return</i>	16.55%	22.96%	39.51%

**Why are the asset allocation effects of asset A and B identical?**

$$AAE (A, 1) \Rightarrow (90\% - 10\%) * (-30\%) = -24\%$$

$$AAE (A, 2) \Rightarrow (10\% - 10\%) * 30\% = 0\%$$

$$AAE (A, 1+2) \Rightarrow (-24\%) * (1 + (-24\%)) + 0\% * (1 + 25\%)$$

$$\Rightarrow -24\% + 5.76\% + 0\% + 0\% = -18.24\%$$

=> It seems that the asset allocation decisions in period 2 did not add any value!

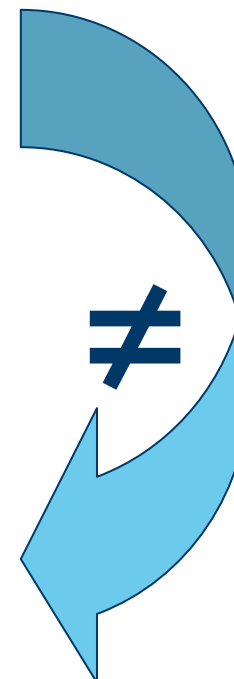
=> The value added is neutralized and moved to another effect! Why!



## TWR-attribution - example without external cash flows (5/6)

Return contributions	Period 1	Period 2	Total
SPE (A)	6.00%	-5.00%	-1.69%
SPE (B)	-45.00%	36.00%	10.80%
<i>Total SPE</i>	-39.00%	31.00%	9.11%
AAE (A)	-24.00%	0.00%	-18.24%
AAE (B)	-24.00%	0.00%	-18.24%
<i>Total AAE</i>	-48.00%	0.00%	-36.48%

Profit contributions	Period 1	Period 2	Total
SPE (A)	9.00	-8.13	0.87
SPE (B)	-67.50	61.70	-5.81
<i>Total SPE</i>	-58.50	53.57	-4.94
AAE (A)	-36.00	-2.16	-38.16
AAE (A)	-36.00	19.44	-16.56
<i>Total AAE</i>	-72.00	17.28	-54.72



**SPE:** Taking benchmark weights and portfolio / benchmark returns => calculation of absolute profit/loss

**AAE:** Taking benchmark / portfolio weights and benchmark returns => calculation of absolute profit/loss

## TWR-attribution - example without external cash flows (6/6)

Some questions to be answered:

- Why are the absolute profit/loss figures not in line with the returns?
- Why do changes in weights have no impact on the management effects?
- Why are the management effects compounded by the benchmark return, the portfolio return or something similar?
- What is the economical interpretation of such a mathematical procedure?

=> With TWRs we neglect implicitly internal cash flows and herewith neutralize their value by moving these contributions to other effects.

# IRR-attribution - some formulas

(1/3)

How to calculate the IRR management effects?

$$\Rightarrow \text{asset allocation effect} = \text{IRR} (w/c_p; r_b) - \text{IRR} (w/c_b; r_b)$$

$$\Rightarrow \text{stock picking effect} = \text{IRR} (w/c_b; r_p) - \text{IRR} (w/c_b; r_b)$$

$$\begin{aligned} \Rightarrow \text{interaction effect} &= [\text{IRR} (w/c_p; r_p) - \text{IRR} (w/c_b; r_p)] \\ &- [\text{IRR} (w/c_p; r_b) - \text{IRR} (w/c_b; r_b)] \end{aligned}$$

e.g.  $\text{IRR} (w/c_p; r_b)$  : IRR using weights (w) and cash flows ( $c_p; c_b$ ) of portfolio and returns of benchmark ( $r_p; r_b$ )

## IRR-attribution - some formulas

(2/3)

$$(12) \text{ PL}_{\text{Total}} = \sum_{i=1}^n \text{ PL}_i$$

$$(13) \text{ AIC}_{\text{Total}} = \frac{\text{ PL}_{\text{Total}}}{\text{ IRR}_{\text{Total}}}$$

$$(14) \text{ AIC}_i = \frac{\text{ PL}_i}{\text{ IRR}_i}$$

$$(15) \text{ IRR}_{\text{Total}} = \sum_{i=1}^n \text{ RC}_i$$

$$(16) \text{ RC}_i = \frac{\text{ AIC}_i}{\text{ AIC}_{\text{Total}}} * \text{ IRR}_i$$

$$(17) \text{ AIC}_{\text{Total}} \neq \sum_{i=1}^n \text{ AIC}_i !$$

$\text{ PL}_{i;\text{Total}}$  = profit / Loss

$\text{ AIC}_{i;\text{Total}}$  = average invested capital

$\text{ IRR}_{i;\text{Total}}$  = internal rate of return

$\text{ RC}_i$  = return contribution

for a single period = for multi-period

# IRR-attribution - some formulas

(3/3)

$$(18) ER_{\text{Total}} = AAE_{\text{Total}} + SPE_{\text{Total}} + IAE_{\text{Total}}$$

$$(19) AAE_{\text{Total}} = \sum_{i=1}^n AAE_i = \sum_{i=1}^n [RC_i(w/c_p; r_b) - RC_i(w/c_b; r_b)]$$

$$(20) SPE_{\text{Total}} = \sum_{i=1}^n SPE_i = \sum_{i=1}^n [RC_i(w/c_b; r_p) - RC_i(w/c_b; r_b)]$$

$$(21) IAE_{\text{Total}} = \sum_{i=1}^n IAE_i = \sum_{i=1}^n \left\{ \begin{bmatrix} RC_i(w/c_p; r_p) \\ -RC_i(w/c_b; r_p) \end{bmatrix} - \begin{bmatrix} RC_i(w/c_p; r_b) \\ -RC_i(w/c_b; r_b) \end{bmatrix} \right\}$$

$ER_{i;\text{Total}}$  = excess return

$AAE_{i;\text{Total}}$  = asset allocation effect

$SPE_{i;\text{Total}}$  = stock picking effect

$IAE_{i;\text{Total}}$  = interaction effect

for a single period = for multi-period

# IRR-attribution - example without external cash flows (1/5)

Portfolio			
	Period 1	Period 2	
Dates	31.12.2007	31.12.2008	31.12.2009
	Cash flow at beginning of period	Cash flow at beginning of period	Investment at the end of period
Asset A	<b>-135.00</b>	<b>156.75</b>	15.00
Asset B	<b>-15.00</b>	<b>-156.75</b>	185.63
Total assets	-150.00	0.00	200.63
	weights at beginning of period	weights at beginning of period	weights at the end of period
Asset A	90.00%	10.00%	7.48%
Asset B	10.00%	90.00%	92.52%
Total assets	100.00%	100.00%	100.00%
	Returns for period	Returns for period	Cummulative Returns (IRR)
Asset A	<b>30.00%</b>	<b>-20.00%</b>	56.17%
Asset B	<b>-20.00%</b>	<b>10.00%</b>	15.34%
Total assets	25.00%	7.00%	33.75%

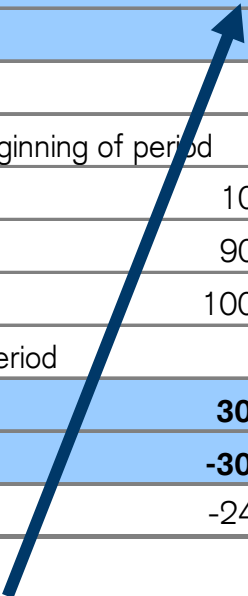


$$156.75 = 135 * (1+30\%) - 150 * (1+25\%) * 10\%$$

no external cash flows

# IRR-attribution - example without external cash flows (2/5)

Benchmark			
	Period 1	Period 2	
Dates	31.12.2007	31.12.2008	31.12.2009
	Cash flow at beginning of period	Cash flow at beginning of period	Investment at the end of period
Asset A	<b>-15.00</b>	<b>-8.10</b>	24.18
Asset B	<b>-135.00</b>	<b>8.10</b>	117.18
Total assets	-150.00	0.00	141.36
	weights at beginning of period	weights at beginning of period	weights at the end of period
Asset A	10.00%	10.00%	17.11%
Asset B	90.00%	90.00%	82.89%
Total assets	100.00%	100.00%	100.00%
	Returns for period	Returns for period	Cummulative Returns (IRR)
Asset A	<b>-30.00%</b>	<b>30.00%</b>	5.69%
Asset B	<b>30.00%</b>	<b>-30.00%</b>	-7.43%
Total assets	24.00%	-24.00%	-5.76%



$$8.10 = 15 * (1-30\%) - 150 * (1+24\%) * 10\%$$

no external cash flows

# IRR-attribution - example without external cash flows (3/5)

<b>IRR(<math>c_p, r_b</math>)</b>	<b>31.12.2003</b>	<b>31.12.2004</b>	<b>31.12.2005</b>	<b>Total Period</b>	<b>Ø Capital</b>	<b>Contributions</b>
A: Weights	<b>1</b> 90.00%	<b>3</b> 10.00%				
A: Assets to invest	135.00	11.40	14.82			
A: Return		<b>2</b> -30.00%	<b>5</b> 30.00%	<b>6</b> -42.21%	<b>87.85</b>	<b>7</b> -24.72%
A: Profit/Loss		<b>4</b> -40.50	3.42	-37.08		-37.08
A: Cash flow	135.00	-83.10				
B: Weights	10.00%	90.00%				
B: Assets to invest	15.00	102.60	71.82			
B: Return		30.00%	-30.00%	-42.27%	<b>62.17</b>	<b>-17.52%</b>
B: Profit/Loss		4.50	-30.78	-26.28		<b>-26.28</b>
B: Cash flow	15.00	83.10				
Total: Weights	100.00%	100.00%	0.00%			
Total: Assets to invest	150.00	114.00	86.64			
Total: Return		-24.00%	-24.00%	-42.24%	<b>150.00</b>	<b>-42.24%</b>
Total: Profit/Loss		-36.00	-27.36	-63.36		<b>-63.36</b>
Total: Cash flow	150.00	0.00	0.00			



# IRR-attribution - example without external cash flows (4/5)

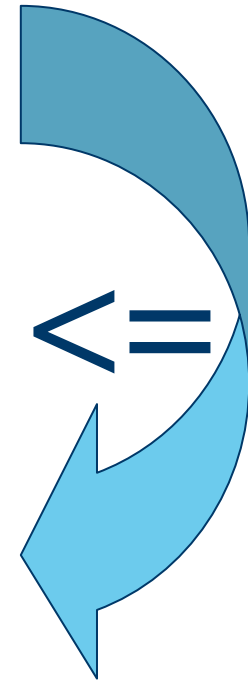
<b>IRR(<math>c_b, r_b</math>)</b>	<b>31.12.2003</b>	<b>31.12.2004</b>	<b>31.12.2005</b>	<b>Total Period</b>	<b>Ø Capital</b>	<b>Contributions</b>
A: Weights	10.00%	10.00%				
A: Assets to invest	15.00	18.60	24.18			
A: Return		-30.00%	30.00%	5.69%	<b>18.99</b>	<b>0.72%</b>
A: Profit/Loss		-4.50	5.58	1.08		<b>1.08</b>
A: Cash flow	15.00	8.10				
<hr/>						
B: Weights	90.00%	90.00%				
B: Assets to invest	135.00	167.40	117.18			
B: Return		30.00%	-30.00%	-7.43%	<b>130.88</b>	<b>-6.48%</b>
B: Profit/Loss		40.50	-50.22	-9.72		<b>-9.72</b>
B: Cash flow	135.000	-8.100				
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Total: Weights	100.00%	100.00%	0.00%			
Total: Assets to invest	150.00	186.00	141.36			
Total: Return		24.00%	-24.00%	-5.76%	<b>150.00</b>	<b>-5.76%</b>
Total: Profit/Loss		36.00	-44.64	-8.64		<b>-8.64</b>
Total: Cash flow	150.00	0.00	0.00			



# IRR-attribution - example without external cash flows (5/5)

<b>Return contributions</b>	<b>Period 1</b>	<b>Period 2</b>	<b>Total</b>
SPE (A)			0.58%
SPE (B)			-3.87%
<i>Total SPE</i>			-3.29%
AAE (A)			-25.44%
AAE (B)			-11.04%
<i>Total AAE</i>			-36.48%

<b>Profit contributions</b>	<b>Period 1</b>	<b>Period 2</b>	<b>Total</b>
SPE (A)	9.00	-8.13	0.87
SPE (B)	-67.50	61.70	-5.81
<i>Total SPE</i>	-58.50	53.57	-4.94
AAE (A)	-36.00	-2.16	-38.16
AAE (A)	-36.00	19.44	-16.56
<i>Total AAE</i>	-72.00	17.28	-54.72



# TWR- versus IRR-attribution - example without cash flows

## TWR-attribution effects

	<b>Asset A</b>	<b>Asset B</b>	<b>Total effect</b>
Asset allocation effect	-18.24%	-18.24%	-36.48%
Stock picking effect	-1.69%	10.80%	9.11%
Interaction effect	36.48%	30.40%	66.88%
Total excess return	16.55%	22.96%	39.51%

## IRR-attribution effects

	<b>Asset A</b>	<b>Asset B</b>	<b>Total effect</b>
Asset allocation effect	-25.44%	-11.04%	-36.48%
Stock picking effect	0.58%	-3.87%	-3.29%
Interaction effect	48.64%	30.64%	79.28%
Total excess return	23.78%	15.73%	39.51%

=> differences in attribution effects can be substantial

# TWR-attribution - example with external cash flows (1/2)

Portfolio			
	Period 1	Period 2	
Dates	31.12.2003	31.12.2004	31.12.2005
	Investment at beginning of period	Investment at beginning of period	Investment at the end of period
Asset A	135.00	28.75	23.00
Asset B	15.00	258.75	284.63
Total assets	150.00	<b>287.50</b>	307.63
	weights at beginning of period	weights at beginning of period	weights at the end of period
Asset A	<b>90.00%</b>	<b>10.00%</b>	7.48%
Asset B	<b>10.00%</b>	<b>90.00%</b>	92.52%
Total assets	100.00%	100.00%	100.00%
	Returns for period	Returns for period	Cummulative Returns (TWR)
Asset A	<b>30.00%</b>	<b>-20.00%</b>	4.00%
Asset B	<b>-20.00%</b>	<b>10.00%</b>	-12.00%
Total assets	25.00%	7.00%	33.75%

with external cash flow at the beginning of period 2 (+100)

# TWR-attribution - example with external cash flows (2/2)

Benchmark			
	Period 1	Period 2	
Dates	31.12.2003	31.12.2004	31.12.2005
	Investment at beginning of period	Investment at beginning of period	Investment at the end of period
Asset A	15.00	28.60	37.18
Asset B	135.00	257.40	180.18
Total assets	150.00	<b>286.00</b>	217.36
	weights at beginning of period	weights at beginning of period	weights at the end of period
Asset A	<b>10.00%</b>	<b>10.00%</b>	17.11%
Asset B	<b>90.00%</b>	<b>90.00%</b>	82.89%
Total assets	100.00%	100.00%	100.00%
	Returns for period	Returns for period	Cummulative Returns (TWR)
Asset A	<b>-30.00%</b>	<b>30.00%</b>	-9.00%
Asset B	<b>30.00%</b>	<b>-30.00%</b>	-9.00%
Total assets	24.00%	-24.00%	-5.76%

with external cash flow at the beginning of period 2 (+100)

# IRR-attribution - example with external cash flows (1/2)

Portfolio			
	Period 1	Period 2	
Dates	31.12.2003	31.12.2004	31.12.2005
	Cash flow at beginning of period	Cash flow at beginning of period	Investment at the end of period
Asset A	-135.00	146.75	23.00
Asset B	-15.00	-246.75	284.63
Total assets	-150.00	-100.00	307.63
	weights at beginning of period	weights at beginning of period	weights at the end of period
Asset A	90.00%	10.00%	7.48%
Asset B	10.00%	90.00%	92.52%
Total assets	100.00%	100.00%	100.00%
	Returns for period	Returns for period	Cummulative Returns (IRR)
Asset A	30.00%	-20.00%	50.24%
Asset B	-20.00%	10.00%	17.16%
Total assets	25.00%	7.00%	29.29%

$$156.75 = 135 * (1+30\%) - (150 * (1+25\%) + 100) * 10\%$$

with external cash flow at the beginning of period 2 (+100)

# IRR-attribution - example with external cash flows (2/2)

Benchmark			
	Period 1	Period 2	
Dates	31.12.2003	31.12.2004	31.12.2005
	Cash flow at beginning of period	Cash flow at beginning of period	Investment at the end of period
Asset A	-15.00	-18.10	37.18
Asset B	-135.00	-81.90	180.18
Total assets	-150.00	-100.00	217.36
	weights at beginning of period	weights at beginning of period	weights at the end of period
Asset A	10.00%	10.00%	17.11%
Asset B	90.00%	90.00%	82.89%
Total assets	100.00%	100.00%	100.00%
	Returns for period	Returns for period	Cummulative Returns (IRR)
Asset A	-30.00%	30.00%	17.23%
Asset B	30.00%	-30.00%	-20.60%
Total assets	24.00%	-24.00%	-16.15%

$$18.10 = 15 * (1-30\%) - (150 * (1+24\%) + 100) * 10\%$$

with external cash flow at the beginning of period 2 (+100)

# TWR- versus IRR-attribution - example with cash flows

## TWR-attribution effects (same as without cash flows)

	<b>Asset A</b>	<b>Asset B</b>	<b>Total effect</b>
Asset allocation effect	-18.24%	-18.24%	-36.48%
Stock picking effect	-1.69%	10.80%	9.11%
Interaction effect	36.48%	30.40%	66.88%
Total excess return	16.55%	22.96%	39.51%

## IRR-attribution effects (different from without cash flows)

	<b>Asset A</b>	<b>Asset B</b>	<b>Total effect</b>
Asset allocation effect	-26.74%	0.65%	-26.09%
Stock picking effect	-0.72%	7.82%	7.10%
Interaction effect	43.10%	21.33%	64.43%
Total excess return	15.65%	29.79%	45.44%



## Case for action -2-



# General framework for decomposing returns

(1/3)

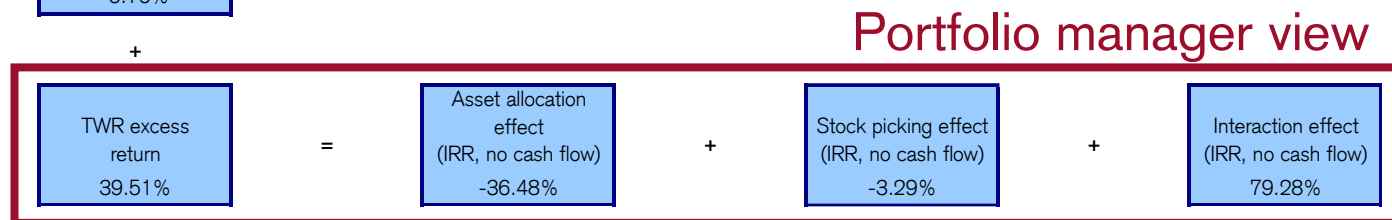
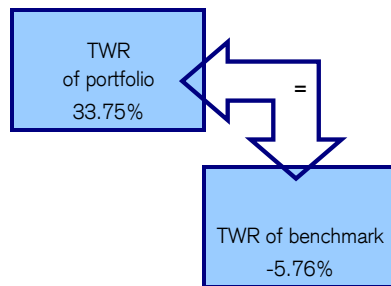
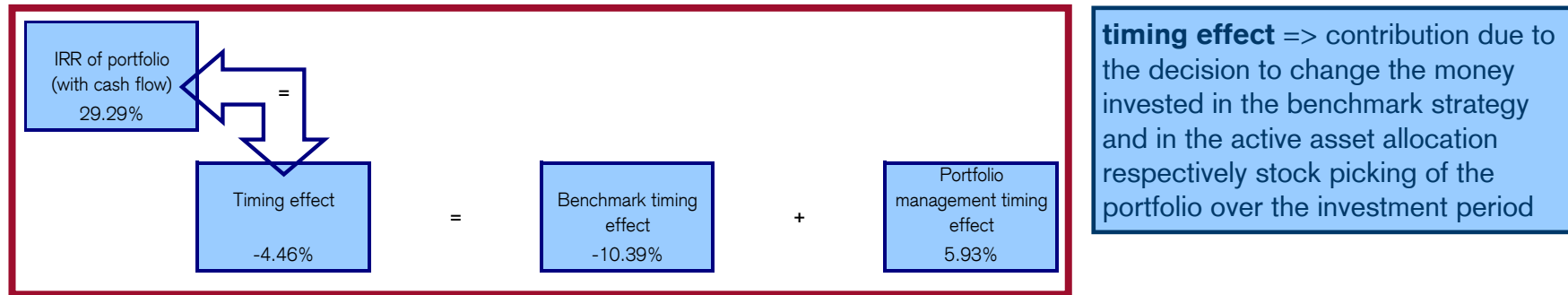
The MWR-calculation and MWR-attribution allow to define a general framework for decomposing returns:

- That combines the different views on performance (client versus portfolio manager).
- That connects the different return measurement methods (TWR and MWR).
- That connects the different return attribution methods (TWR and MWR).
- That corresponds to absolute profit & loss measurement and profit & loss attribution.
- Etc.

# General framework for decomposing returns

(2/3)

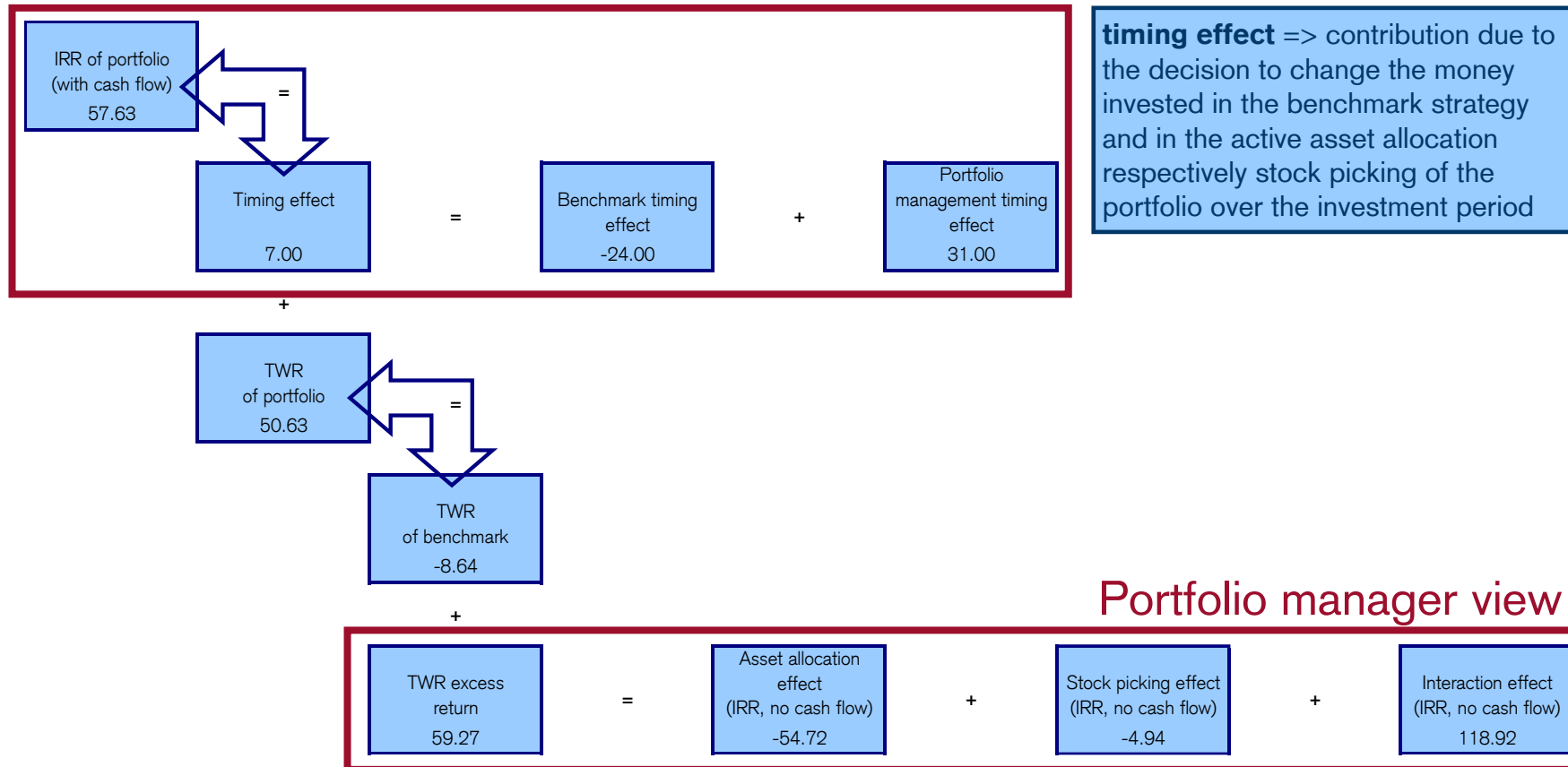
## Client view



# General framework for decomposing returns

(3/3)

## Client view



# Simple example for an IRR implementation

(1/2)

Asset Class	Asset at 31.12.08	Sum of cash flows	Assets at 30.12.2009	Profit & loss (in CHF)	Profit & loss (in %)	Sum of cash flows in % of assets at 31.12.2008	On average invested capital (in CHF)	On average invested capital (in %)
Cash	77'299'536	-44'553'165	37'028'584	4'282'214	6.77%	-57.64%	64'195'773	8.50%
Bonds CHF	90'532'962	-14'375'791	80'742'922	4'585'751	7.25%	-15.88%	89'041'307	11.79%
Convertibles	11'989'017	12'524'530	28'169'285	3'655'738	5.78%	104.47%	19'773'155	2.62%
Mortages (direct)	88'682'597	-2'828'500	88'446'889	2'592'792	4.10%	-3.19%	86'973'765	11.52%
Mortages (indirect)	78'401'686	-11'744'126	68'594'225	1'936'665	3.06%	-14.98%	71'637'225	9.49%
Swiss Equities	88'308'678	15'142'362	119'931'920	16'480'880	26.04%	17.15%	91'359'125	12.10%
Foreign Equities	82'870'803	37'716'722	141'375'901	20'788'376	32.85%	45.51%	92'020'100	12.19%
Swiss Real Estate	204'598'112	-26'089'690	189'673'326	11'164'904	17.64%	-12.75%	193'225'914	25.59%
Foreign Real Estate	19'409'158	-4'485'424	13'162'380	-1'761'354	-2.78%	-23.11%	15'751'996	2.09%
Hedge Funds	17'680'187	35'756'596	52'996'707	-440'076	-0.70%	202.24%	30'541'714	4.05%
<b>Total</b>	<b>759'772'736</b>	<b>-2'936'486</b>	<b>820'122'139</b>	<b>63'285'889</b>	<b>100.00%</b>	<b>-0.39%</b>	<b>755'014'682</b>	<b>100.00%</b>

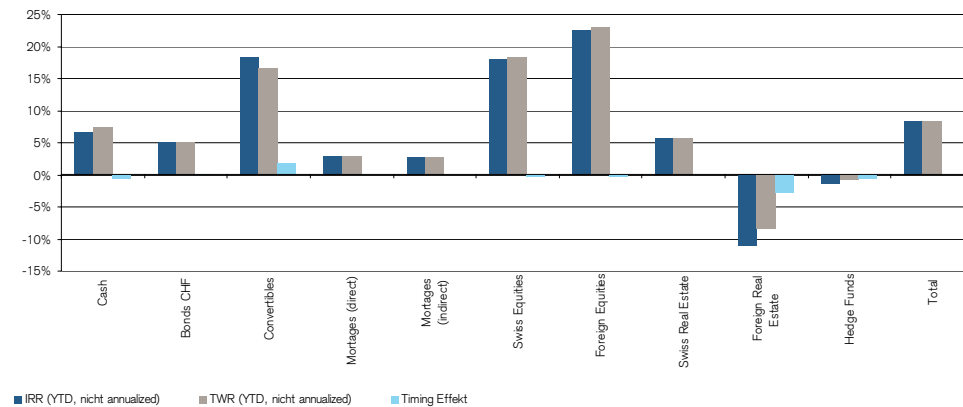
- Data needed:**
- TWRs and TWR contributions
  - Month end valuations
  - Cash flows during the month

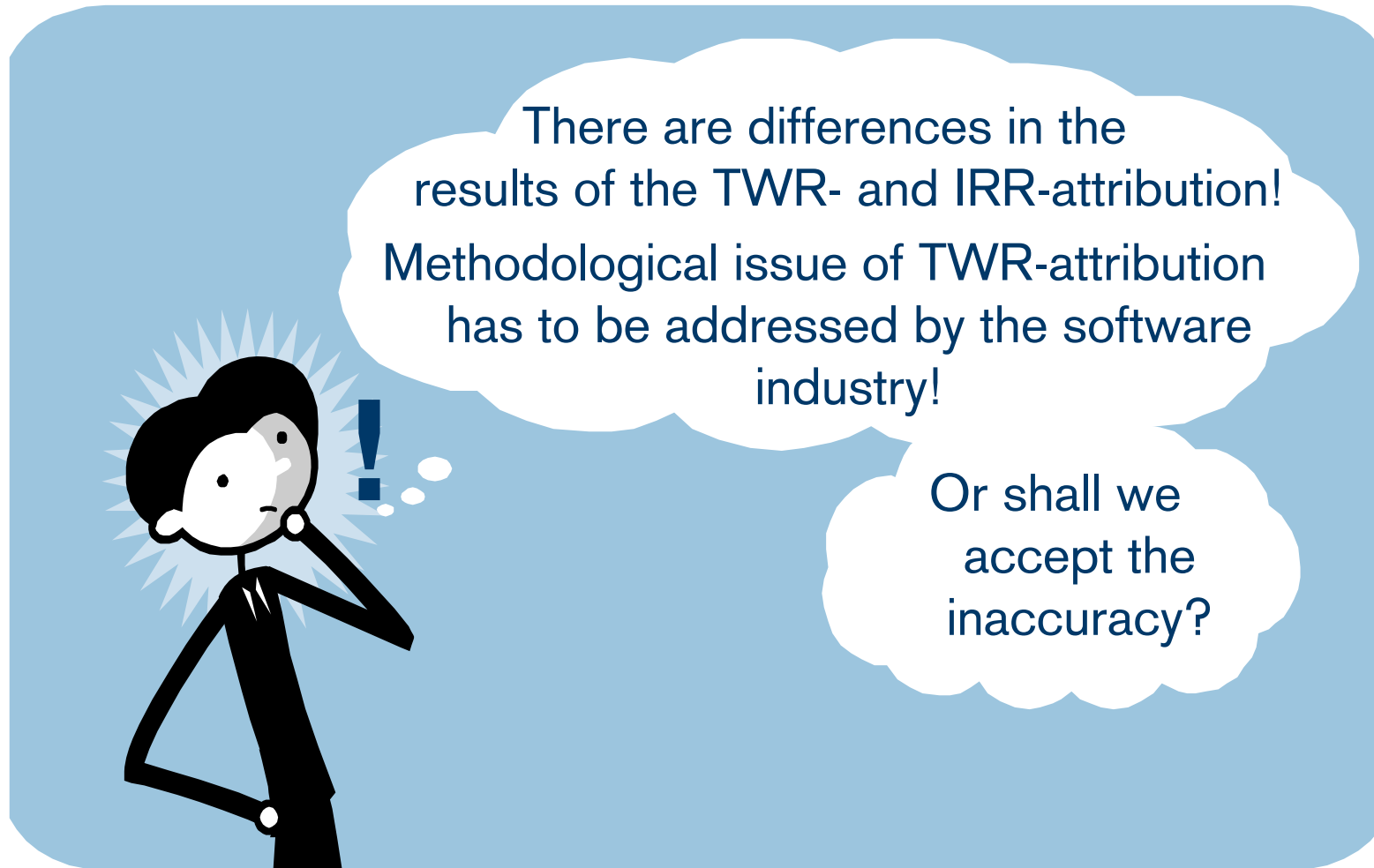
# Simple example for an IRR implementation

(2/2)

Asset Class	Profit & loss (in CHF)	IRR		TWR	
		(YTD, nicht annualized)	IRR Contribution	(YTD, nicht annualized)	
Cash	4'282'214	6.67%	0.57%	-0.69%	7.36%
Bonds CHF	4'585'751	5.15%	0.61%	-0.01%	5.16%
Convertibles	3'655'738	18.49%	0.48%	1.89%	16.60%
Mortages (direct)	2'592'792	2.98%	0.34%	0.01%	2.97%
Mortages (indirect)	1'936'665	2.70%	0.26%	0.00%	2.70%
Swiss Equities	16'480'880	18.04%	2.18%	-0.44%	18.48%
Foreign Equities	20'788'376	22.59%	2.75%	-0.44%	23.03%
Swiss Real Estate	11'164'904	5.78%	1.40%	0.01%	5.74%
Foreign Real Estate	-1'761'354	-11.18%			
Hedge Funds	-440'076	-1.44%			
<b>Total</b>	<b>63'285'889</b>	<b>8.38%</b>			

IRR versus TWR





# Conclusions

(2/3)

"Odd" questions can be better answered using the general framework for decomposing returns; including MWRs!

The return is positive but I lost money - how come?

Multiplying the weights with the return does not lead to my absolute profit - how come?

The segment return is positive but its return contribution is negative - how come?

What is my on average invested capital?



# Conclusions

(3/3)

- Asset Managers should start thinking of including the "MWR-concept" in their performance evaluations.
- Software firms should start thinking of including the "MWR-concept" in their performance attribution methodology.
- ...

## Appendix - another example

(1/9)

- 3 investment periods (for example months or days).
- 4 asset classes.
- Portfolio manager has discretion over the internal cash flows.
- MWR is calculated using the IRR over the entire investment period.
- TWR- and IRR-attribution follows the methods explained before.
- Portfolio manager does asset allocation decisions.
- 2 cases: 1) without external cash flows and 2) with external cash flows by the client at the beginning of period 1 and 2.
- If, external cash flows are invested according to the current asset allocation.

# Appendix - another example

(2/9)

## Input data: returns

	Return Period 1	Case 1 Return Period 2	Return Period 3	Return Period 1	Case 2 Return Period 2	Return Period 3
<b>Asset Class A</b>						
Portfolio	0.60%	0.80%	1.00%	0.60%	0.80%	1.00%
Benchmark	0.40%	0.50%	0.30%	0.40%	0.50%	0.30%
<i>Difference</i>	0.20%	0.30%	0.70%	0.20%	0.30%	0.70%
<b>Asset Class B</b>						
Portfolio	-2.00%	-3.00%	-2.50%	-2.00%	-3.00%	-2.50%
Benchmark	-1.00%	-1.60%	-0.80%	-1.00%	-1.60%	-0.80%
<i>Difference</i>	-1.00%	-1.40%	-1.70%	-1.00%	-1.40%	-1.70%
<b>Asset Class C</b>						
Portfolio	6.00%	4.20%	7.00%	6.00%	4.20%	7.00%
Benchmark	6.60%	5.00%	8.00%	6.60%	5.00%	8.00%
<i>Difference</i>	-0.60%	-0.80%	-1.00%	-0.60%	-0.80%	-1.00%
<b>Asset Class D</b>						
Portfolio	1.00%	2.00%	1.40%	1.00%	2.00%	1.40%
Benchmark	2.00%	3.00%	2.40%	2.00%	3.00%	2.40%
<i>Difference</i>	-1.00%	-1.00%	-1.00%	-1.00%	-1.00%	-1.00%

# Appendix - another example

(3/9)

## Input data: asset class weights

	Weight Beginning of Period 1	Case 1 Weight Beginning of Period 2	Weight Beginning of Period 3	Weight Beginning of Period 1	Case 2 Weight Beginning of Period 2	Weight Beginning of Period 3
<b>Asset Class A</b>						
Portfolio	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Benchmark	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
<i>Difference</i>	-10.00%	-10.00%	-10.00%	-10.00%	-10.00%	-10.00%
<b>Asset Class B</b>						
Portfolio	30.00%	60.00%	70.00%	30.00%	60.00%	70.00%
Benchmark	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%
<i>Difference</i>	-10.00%	20.00%	30.00%	-10.00%	20.00%	30.00%
<b>Asset Class C</b>						
Portfolio	50.00%	20.00%	10.00%	50.00%	20.00%	10.00%
Benchmark	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%
<i>Difference</i>	25.00%	-5.00%	-15.00%	25.00%	-5.00%	-15.00%
<b>Asset Class D</b>						
Portfolio	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Benchmark	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
<i>Difference</i>	-5.00%	-5.00%	-5.00%	-5.00%	-5.00%	-5.00%

# Appendix - another example

(4/9)

## Input data: beginning period cash flows

	Case 1			Case 2		
	Cash Flow Beginning of Period 1	Cash Flow Beginning of Period 2	Cash Flow Beginning of Period 3	Cash Flow Beginning of Period 1	Cash Flow Beginning of Period 2	Cash Flow Beginning of Period 3
<b>Asset Class A</b>						
Portfolio	10.00	0.20	-0.15	10.00	5.20	-3.23
Benchmark	20.00	0.25	0.13	20.00	10.25	-5.80
<i>Difference</i>	-10.00	-0.05	-0.29	-10.00	-5.05	2.57
<b>Asset Class B</b>						
Portfolio	30.00	32.14	11.61	30.00	62.14	-3.72
Benchmark	40.00	1.05	1.12	40.00	21.05	-10.33
<i>Difference</i>	-10.00	31.08	10.49	-10.00	41.08	6.60
<b>Asset Class C</b>						
Portfolio	50.00	-32.49	-11.19	50.00	-22.49	-19.64
Benchmark	25.00	-1.24	-0.98	25.00	11.26	-8.96
<i>Difference</i>	25.00	-31.25	-10.21	25.00	-33.75	-10.69
<b>Asset Class D</b>						
Portfolio	10.00	0.16	-0.27	10.00	5.16	-3.41
Benchmark	15.00	-0.06	-0.28	15.00	7.44	-4.92
<i>Difference</i>	-5.00	0.21	0.01	-5.00	-2.29	1.51
<b>Total</b>						
Portfolio	100.00	0.00	0.00	100.00	50.00	-30.00
Benchmark	100.00	0.00	0.00	100.00	50.00	-30.00
<i>Difference</i>	0.00	0.00	0.00	0.00	0.00	0.00

(internal)  
cash flows  
even if  
there are  
no external  
cash flows

# Appendix - another example

(5/9)

## Return figures: overview

	Return Portfolio	Return Benchmark	Excess Return	Asset Allocation Effect	Stock Picking Effect	Interaction Effect	Total Effects	Difference of Value Added
<b>Case 1</b>								
TWR Attribution	1.04%	4.97%	-3.93%	-0.76%	-2.53%	-0.64%	-3.93%	0.00000%
IRR Attribution	1.04%	4.97%	-3.93%	-0.80%	-2.51%	-0.62%	-3.93%	0.00000%
<i>Difference</i>	0.00000%	0.00000%	0.00000%	0.03365%	-0.01925%	-0.01440%	0.00000%	0.00000%
<b>Case 2</b>								
TWR Attribution	1.04%	4.97%	-3.93%	-0.76%	-2.53%	-0.64%	-3.93%	0.00000%
IRR Attribution	0.44%	4.85%	-4.42%	-1.23%	-2.54%	-0.65%	-4.42%	0.00000%
<i>Difference</i>	0.60104%	0.11500%	0.48605%	0.46384%	0.01456%	0.00765%	0.48605%	0.00000%

**Case 1:** TWR is identical to the IRR on the total asset level as there are no external cash flows, but management effects are different.

**Case 2:** TWR and IRR are different on all asset levels and for all management effects.

# Appendix - another example

(6/9)

## Return figures: details for case 1

		Return Portfolio	Contribution to Portfolio Return	Return Benchmark	Contribution to Benchmark Return	Excess Return
<b>Case 1</b>						
TWR Attribution	Asset Class A	2.42%	0.24%	1.20%	0.24%	-0.01%
	Asset Class B	-7.32%	-4.13%	-3.36%	-1.39%	-2.74%
	Asset Class C	18.18%	4.49%	20.88%	4.98%	-0.49%
	Asset Class D	4.46%	0.44%	7.58%	1.13%	-0.69%
	<i>Total</i>	1.04%	1.04%	4.97%	4.97%	-3.93%
IRR Attribution	Asset Class A	2.42%	0.24%	1.20%	0.24%	0.00%
	Asset Class B	-7.59%	-4.23%	-3.36%	-1.38%	-2.85%
	Asset Class C	18.00%	4.57%	20.83%	4.98%	-0.40%
	Asset Class D	4.47%	0.45%	7.58%	1.13%	-0.68%
	<i>Total</i>	1.04%	1.04%	4.97%	4.97%	-3.93%
Differences	Asset Class A	0.00%	-0.01%	0.00%	0.00%	-0.01%
	Asset Class B	0.28%	0.10%	-0.01%	-0.01%	0.11%
	Asset Class C	0.19%	-0.09%	0.06%	0.00%	-0.09%
	Asset Class D	-0.01%	-0.01%	0.00%	0.00%	-0.01%
	<i>Total</i>	0.00%	0.00%	0.00%	0.00%	0.00%



TWR- and IRR-contributions are not identical due to internal cash flows.

# Appendix - another example

(7/9)

## Return figures: details for case 2

		Contribution to Portfolio Return		Contribution to Benchmark Return		Excess Return
		Return Portfolio	Portfolio Return	Return Benchmark	Benchmark Return	
<b>Case 2</b>						
TWR Attribution	Asset Class A	2.42%	0.24%	1.20%	0.24%	-0.01%
	Asset Class B	-7.32%	-4.13%	-3.36%	-1.39%	-2.74%
	Asset Class C	18.18%	4.49%	20.88%	4.98%	-0.49%
	Asset Class D	4.46%	0.44%	7.58%	1.13%	-0.69%
	<i>Total</i>	1.04%	1.04%	4.97%	4.97%	-3.93%
IRR Attribution	Asset Class A	2.45%	0.25%	1.23%	0.25%	0.00%
	Asset Class B	-7.74%	-4.44%	-3.49%	-1.43%	-3.01%
	Asset Class C	17.56%	4.16%	20.41%	4.88%	-0.72%
	Asset Class D	4.68%	0.47%	7.79%	1.16%	-0.69%
	<i>Total</i>	0.44%	0.44%	4.85%	4.85%	-4.42%
Differences	Asset Class A	-0.03%	-0.01%	-0.02%	0.00%	0.00%
	Asset Class B	0.42%	0.31%	0.12%	0.05%	0.27%
	Asset Class C	0.62%	0.33%	0.48%	0.10%	0.23%
	Asset Class D	-0.22%	-0.03%	-0.21%	-0.03%	0.00%
	<i>Total</i>	0.60%	0.60%	0.11%	0.11%	0.49%



TWR- and IRR-contributions are not identical on all asset levels.



# Appendix - another example

(8/9)

## Return figures: details for case 1

		Contribution to Asset Allocation Effect	Contribution to Stock Picking Effect	Contribution to Interaction Effect	Contribution to Total Effects	Difference of Value Added
<b>Case 1</b>						
TWR Attribution	Asset Class A	-0.12%	0.25%	-0.12%	0.00%	-0.00541%
	Asset Class B	-0.48%	-1.69%	-0.71%	-2.88%	0.13795%
	Asset Class C	0.22%	-0.62%	0.04%	-0.36%	-0.13258%
	Asset Class D	-0.38%	-0.46%	0.15%	-0.69%	0.00004%
	<i>Total</i>	-0.76%	-2.53%	-0.64%	-3.93%	0.00000%
IRR Attribution	Asset Class A	-0.12%	0.24%	-0.12%	0.00%	0.00000%
	Asset Class B	-0.49%	-1.65%	-0.71%	-2.85%	0.00000%
	Asset Class C	0.18%	-0.64%	0.06%	-0.40%	0.00000%
	Asset Class D	-0.37%	-0.46%	0.15%	-0.68%	0.00000%
	<i>Total</i>	-0.80%	-2.51%	-0.62%	-3.93%	0.00000%
Differences	Asset Class A	0.00%	0.01%	0.00%	0.00%	-0.00541%
	Asset Class B	0.02%	-0.05%	0.00%	-0.03%	0.13795%
	Asset Class C	0.04%	0.02%	-0.02%	0.04%	-0.13258%
	Asset Class D	-0.01%	0.00%	0.00%	-0.01%	0.00004%
	<i>Total</i>	0.03%	-0.02%	-0.01%	0.00%	0.00000%

Effect of the reinvestment assumption

<= no problem



TWR- and IRR-management effects on asset class level are different due to internal cash flows.

# Appendix - another example

(9/9)

## Return figures: details for case 2

		Contribution to Asset Allocation Effect	Contribution to Stock Picking Effect	Contribution to Interaction Effect	Contribution to Total Effects	Difference of Value Added
<b>Case 2</b>						
TWR Attribution	Asset Class A	-0.12%	0.25%	-0.12%	0.00%	-0.00541%
	Asset Class B	-0.48%	-1.69%	-0.71%	-2.88%	0.13795%
	Asset Class C	0.22%	-0.62%	0.04%	-0.36%	-0.13258%
	Asset Class D	-0.38%	-0.46%	0.15%	-0.69%	0.00004%
	<i>Total</i>	-0.76%	-2.53%	-0.64%	-3.93%	0.00000%
IRR Attribution	Asset Class A	-0.12%	0.24%	-0.12%	0.00%	0.00000%
	Asset Class B	-0.57%	-1.67%	-0.77%	-3.01%	0.00000%
	Asset Class C	-0.16%	-0.65%	0.09%	-0.72%	0.00000%
	Asset Class D	-0.38%	-0.46%	0.15%	-0.69%	0.00000%
	<i>Total</i>	-1.23%	-2.54%	-0.65%	-4.42%	0.00000%
Differences	Asset Class A	0.00%	0.00%	0.00%	0.00%	-0.00541%
	Asset Class B	0.09%	-0.02%	0.06%	0.13%	0.13795%
	Asset Class C	0.38%	0.03%	-0.05%	0.36%	-0.13258%
	Asset Class D	0.00%	0.00%	0.00%	0.00%	0.00004%
	<i>Total</i>	0.46%	0.01%	0.01%	0.49%	0.00000%

Effect of the reinvestment assumption

<= no problem



TWR- and IRR-management effects are different on all asset levels.

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