

Overview of Performance Attribution

Version 2.0

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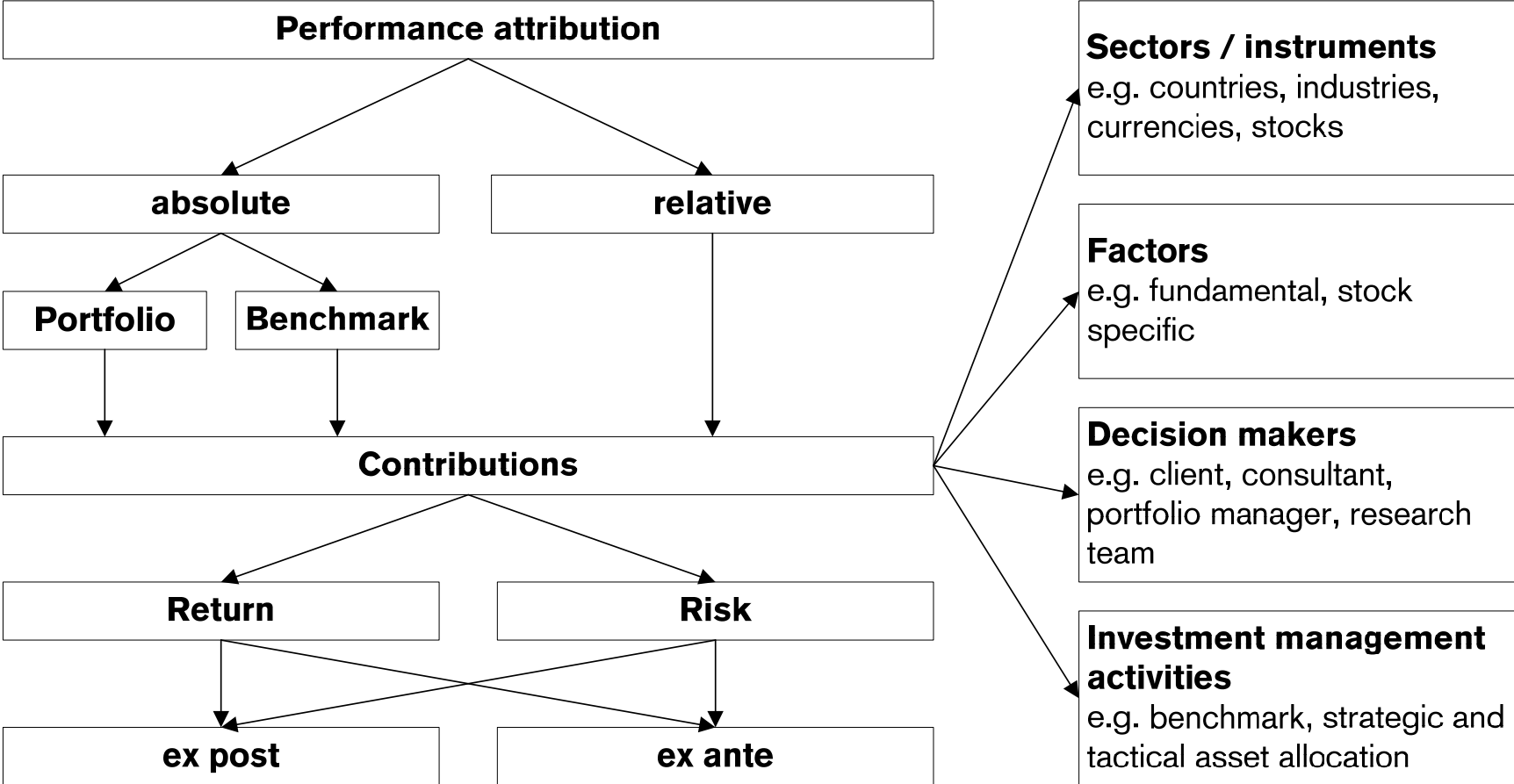
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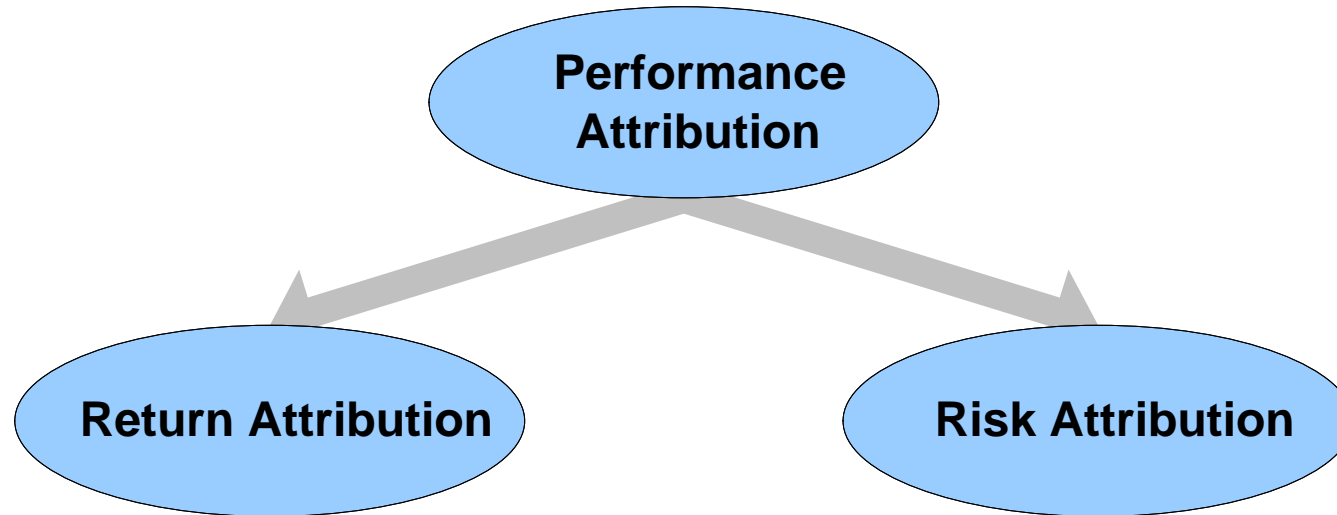
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1. Introduction to return attribution

Performance attribution: the big picture



Return attribution as part of performance attribution



Groups of FT Sector	Asset Allocation	Stock Selection	Interaction	Total
Basic Industries	-0.07%	-0.04%	0.01%	0.04%
Cyclical Consumer Goods	-0.21%	0.26%	-0.11%	-0.06%
Cyclical Services	-0.05%	0.00%	-0.09%	-0.13%
Finance	0.00%	0.11%	0.00%	0.11%
General Industries	0.10%	-0.19%	-0.14%	-0.22%
Information Tech	0.23%	-0.36%	0.08%	-0.07%
Non Cyclical Cons Goods	-0.28%	-0.15%	0.01%	-0.41%
Non Cyclical Services	-0.38%	-0.13%	0.00%	-0.51%
Resources	-0.22%	-0.02%	0.00%	-0.24%
Utilities	-0.13%	0.00%	-0.01%	-0.14%
Other Assets	0.08%	0.00%	0.01%	0.09%
Total	-0.73%	-0.52%	-0.25%	-1.56%

Risk Model : Global	Portfolio	Benchmark
Number of Securities	99	576
Number of Currencies	5	0
Portfolio Value	227'447'728	
Total Risk (ex-ante)	15.76%	15.31%
- Factor Specific Risk	15.53%	15.20%
- Stock Specific Risk	2.72%	1.83%
Tracking Error (ex-ante)	2.35%	
Relative Value at Risk	10'878'425	
R-squared	0.98	
Beta-adjusted Risk	15.59%	15.31%
Predicted Beta	1.02	
Predicted Dividend Yield	2.22	2.37
P/E Ratio (E: 12 months)	38.19	29.42
P/B Ratio (B: year-end)	5.34	4.68

Risk Model : Global	Portfolio	Tracking Error
Total Risk (ex-ante)	15.76%	2.35%
Factor Specific Risk	15.53%	1.40%
- Region	12.40%	0.19%
- Country	9.06%	1.13%
- Industry	3.07%	0.72%
- Fundamental	1.10%	0.48%
- Currency	4.36%	0.52%
- Covariance (+/-)	4.93%	0.60%
Stock Specific Risk	2.72%	1.89%

Definition of return attribution

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- Return attribution is the measurement and quantification of the historical as well as expected return contributions of the individual steps of the investment process as well as of the applied financial instruments.
- We distinguish between return contribution and attribution, whereby return contribution is a more or less arbitrary breakdown of the return using a given breakdown of the investment universe and return attribution is a decision oriented decomposition of the return.
- We distinguish between single factor based attribution and multi factor based attribution where the former approach is mainly used for equity and multi asset class portfolios and the latter approach is used mainly for fixed income portfolios.

Definition of return attribution

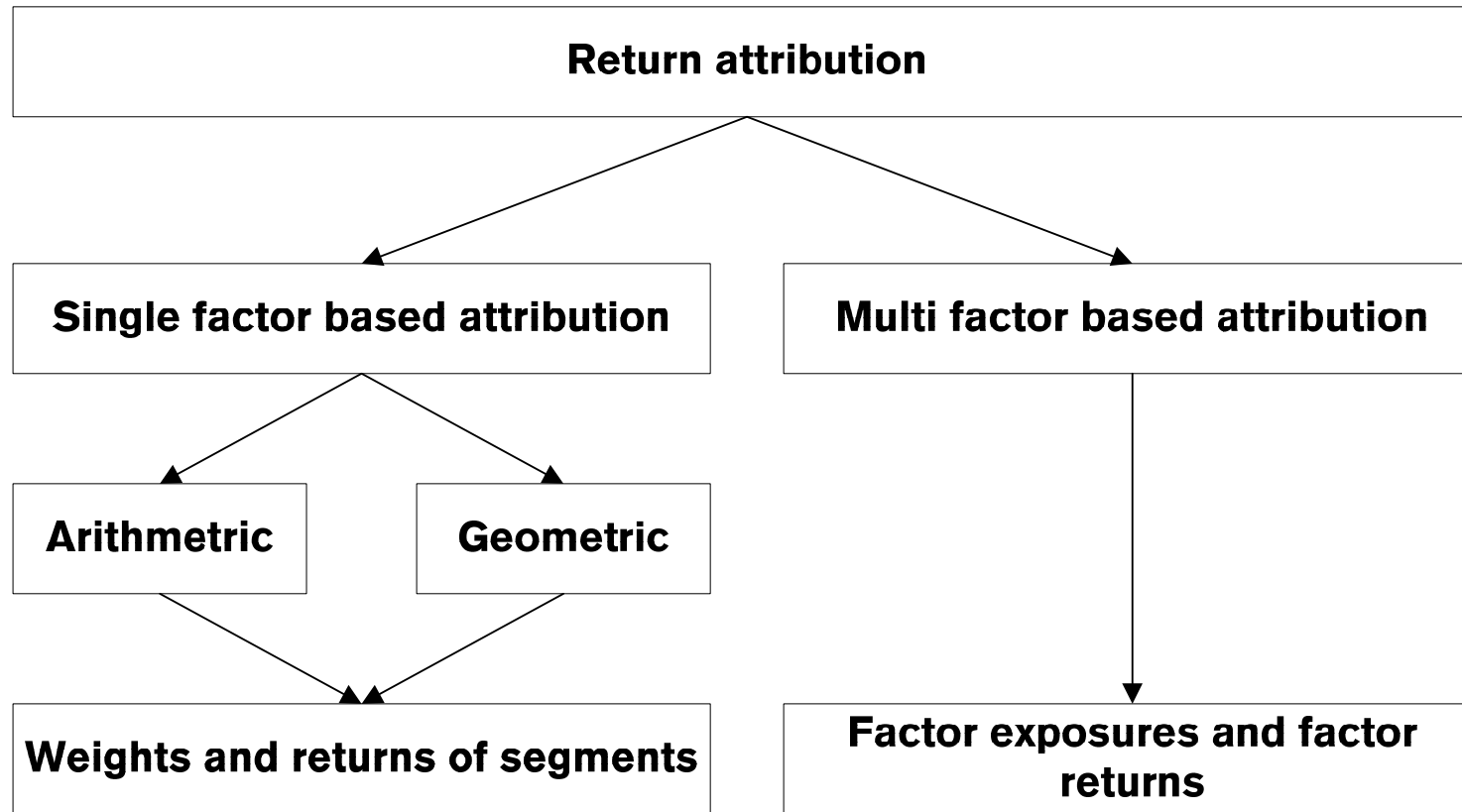
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- We distinguish between arithmetic and geometric attribution where the former explains the excess return as absolute profit expressed as a percentage of the initial amount invested or as an absolute return difference and the latter explains the excess return as absolute profit expressed as a percentage of the final value of the benchmark or as a relative return difference.

Purpose of return attribution

1. Measurement of return and sources of return.
2. Explain on what decisions the return and the sources of return are based on.
3. Support of portfolio management decisions.
4. Monitoring the implementation of the investment strategy.
5. Objectification of the discussions on returns.

Return attribution: the big picture



Remark: In the following we focus on the arithmetic single factor based attribution.

Return attribution: necessary inputs

- Weights of the different asset classes or asset segments for the portfolio for **each** period.
- Weights of the different asset classes or asset segments for the benchmark for **each** period.
- Return of the different asset classes or asset segments for the portfolio for **each** period.
- Return of the different asset classes or asset segments for the benchmark for **each** period.

Contribution to return: single period

$$R^P = \sum_{i=1}^n CR_i^P$$

$$CR_i^P = w_i^P * R_i^P$$

R^P = Portfolioreturn

CR_i^P = Contribution of security i to portfolio return

w_i^P = Beginningweight of security i in portfolio

R_i^P = Return of security i in portfolio

$$R^{BM} = \sum_{i=1}^n CR_i^{BM}$$

$$CR_i^{BM} = w_i^{BM} * R_i^{BM}$$

R^{BM} = Benchmarkreturn

CR_i^{BM} = Contribution of security i to benchmark return

w_i^{BM} = Beginningweight of security i in benchmark

R_i^{BM} = Return of security i in benchmark

$$ER = R^P - R^{BM} = \sum_{i=1}^n (CR_i^P - CR_i^{BM})$$

ER = Excess return

Contribution to return: example for single period

	Portfolio			Benchmark						Total
	Performance	Weight	Contribution	Performance	Weight	Contribution				
Cash	0.50%	10.00%	0.05%	0.10%	6.00%	0.01%				0.04%
Domestic Bonds	1.00%	22.00%	0.22%	3.00%	20.00%	0.60%				-0.38%
Foreign Bonds	3.00%	10.00%	0.30%	2.00%	15.00%	0.30%				0.00%
Domestic Equities	-0.50%	17.00%	-0.09%	-0.10%	14.00%	-0.01%				-0.07%
Foreign Equities	2.00%	12.00%	0.24%	4.00%	16.00%	0.64%				-0.40%
Mortgages	5.00%	5.00%	0.25%	0.50%	3.00%	0.02%				0.24%
Real Estate	1.00%	16.00%	0.16%	2.00%	20.00%	0.40%				-0.24%
Commodities	-12.00%	4.00%	-0.48%	-10.00%	4.00%	-0.40%				-0.08%
Private Equity	2.00%	3.00%	0.06%	1.00%	1.00%	0.01%				0.05%
Hedge Funds	1.00%	1.00%	0.01%	3.00%	1.00%	0.03%				-0.02%
Total	0.73%	100.00%	0.73%	1.59%	100.00%	1.59%				-0.86%

Remark: Contributions may be calculated for any segments or decision but should reflect the investment process.

Underlying decision making process

It is common practice to assume the following **three step** decision making process and to decompose the excess return accordingly

1. Step: Benchmark selection (weights of asset classes).
2. Step: Asset allocation (over- and underweighting of asset classes).
3. Step: Stock picking (over- and underweighting of stocks within asset classes).

Remark: The analysis can be adjusted to any decision making process => see e.g. portfolio attribution.

Contribution to excess return

- **Asset allocation effect**

Contribution to excess return due to the over- and underweighting of asset classes.

- **Stock picking effect**

Contribution to excess return due to the over- and underweighting of stocks within asset classes.

- **Interaction effect**

Contribution to excess return due to the over- and underweighting of out- or underperforming asset classes.

Remark: Contributions may be calculated for any segments or decision but should reflect the investment process.

Contribution to excess return: single period

(1/2)

$$\begin{aligned}ER &= R^P - R^{BM} \\ER &= AAE^{Total} + SPE^{Total} + IAE^{Total} \\ER &= \sum_{i=1}^n AAE_i + \sum_{i=1}^n SPE_i + \sum_{i=1}^n IAE_i \\ER_i &= AAE_i + SPE_i + IAE_i \\AAE^{Total} &= \sum_{i=1}^n AAE_i = \sum_{i=1}^n (w_i^P - w_i^{BM}) * R_i^{BM} \\SPE^{Total} &= \sum_{i=1}^n SPE_i = \sum_{i=1}^n (R_i^P - R_i^{BM}) * w_i^{BM} \\IAE^{Total} &= \sum_{i=1}^n IAE_i = \sum_{i=1}^n [(w_i^P - w_i^{BM}) * (R_i^P - R_i^{BM})]\end{aligned}$$

Remark: There are different ways for calculating the management effects. Here the methodology of Brinson, Hood and Beebower was used.

Contribution to excess return: single period

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AAE^{Total}	=	Total asset allocation effect
SPE^{Total}	=	Total stock picking effect
IAE^{Total}	=	Total interaction effect
AAE_i	=	Asset allocation effect of asset class i
SPE_i	=	Stock picking effect of asset class i
IAE_i	=	Interaction effect of asset class i
ER_i	=	Excess return of asset class i

Starting point: example for single period

	Portfolio			Benchmark			Management Effects			Total
	Performance	Weight	Contribution	Performance	Weight	Contribution	Asset allocation	Stock picking	Interaction	
Cash	0.10%	6.00%	0.01%	0.10%	6.00%	0.01%	0.00%	0.00%	0.00%	0.00%
Domestic Bonds	3.00%	20.00%	0.60%	3.00%	20.00%	0.60%	0.00%	0.00%	0.00%	0.00%
Foreign Bonds	2.00%	15.00%	0.30%	2.00%	15.00%	0.30%	0.00%	0.00%	0.00%	0.00%
Domestic Equities	-0.10%	14.00%	-0.01%	-0.10%	14.00%	-0.01%	0.00%	0.00%	0.00%	0.00%
Foreign Equities	4.00%	16.00%	0.64%	4.00%	16.00%	0.64%	0.00%	0.00%	0.00%	0.00%
Mortgages	0.50%	3.00%	0.02%	0.50%	3.00%	0.02%	0.00%	0.00%	0.00%	0.00%
Real Estate	2.00%	20.00%	0.40%	2.00%	20.00%	0.40%	0.00%	0.00%	0.00%	0.00%
Commodities	-10.00%	4.00%	-0.40%	-10.00%	4.00%	-0.40%	0.00%	0.00%	0.00%	0.00%
Private Equity	1.00%	1.00%	0.01%	1.00%	1.00%	0.01%	0.00%	0.00%	0.00%	0.00%
Hedge Funds	3.00%	1.00%	0.03%	3.00%	1.00%	0.03%	0.00%	0.00%	0.00%	0.00%
Total	1.59%	100.00%	1.59%	1.59%	100.00%	1.59%	0.00%	0.00%	0.00%	0.00%

Remark: Here no bets, everything is indexed.

Asset allocation effect: example for single period

	Portfolio			Benchmark			Management Effects			Total
	Performance	Weight	Contribution	Performance	Weight	Contribution	Asset allocation	Stock picking	Interaction	
Cash	0.10%	10.00%	0.01%	0.10%	6.00%	0.01%	0.00%	0.00%	0.00%	0.00%
Domestic Bonds	3.00%	22.00%	0.66%	3.00%	20.00%	0.60%	0.06%	0.00%	0.00%	0.06%
Foreign Bonds	2.00%	10.00%	0.20%	2.00%	15.00%	0.30%	-0.10%	0.00%	0.00%	-0.10%
Domestic Equities	-0.10%	17.00%	-0.02%	-0.10%	14.00%	-0.01%	0.00%	0.00%	0.00%	0.00%
Foreign Equities	4.00%	12.00%	0.48%	4.00%	16.00%	0.64%	-0.16%	0.00%	0.00%	-0.16%
Mortgages	0.50%	5.00%	0.03%	0.50%	3.00%	0.02%	0.01%	0.00%	0.00%	0.01%
Real Estate	2.00%	16.00%	0.32%	2.00%	20.00%	0.40%	-0.08%	0.00%	0.00%	-0.08%
Commodities	-10.00%	4.00%	-0.40%	-10.00%	4.00%	-0.40%	0.00%	0.00%	0.00%	0.00%
Private Equity	1.00%	3.00%	0.03%	1.00%	1.00%	0.01%	0.02%	0.00%	0.00%	0.02%
Hedge Funds	3.00%	1.00%	0.03%	3.00%	1.00%	0.03%	0.00%	0.00%	0.00%	0.00%
Total	1.34%	100.00%	1.34%	1.59%	100.00%	1.59%	-0.25%	0.00%	0.00%	-0.25%

Remark: Here portfolio returns equals the benchmark return.

Stock picking effect: example for single period

	Portfolio			Benchmark			Management Effects			Total
	Performance	Weight	Contribution	Performance	Weight	Contribution	Asset allocation	Stock picking	Interaction	
Cash	0.50%	6.00%	0.03%	0.10%	6.00%	0.01%	0.00%	0.02%	0.00%	0.02%
Domestic Bonds	1.00%	20.00%	0.20%	3.00%	20.00%	0.60%	0.00%	-0.40%	0.00%	-0.40%
Foreign Bonds	3.00%	15.00%	0.45%	2.00%	15.00%	0.30%	0.00%	0.15%	0.00%	0.15%
Domestic Equities	-0.50%	14.00%	-0.07%	-0.10%	14.00%	-0.01%	0.00%	-0.06%	0.00%	-0.06%
Foreign Equities	2.00%	16.00%	0.32%	4.00%	16.00%	0.64%	0.00%	-0.32%	0.00%	-0.32%
Mortgages	5.00%	3.00%	0.15%	0.50%	3.00%	0.02%	0.00%	0.14%	0.00%	0.14%
Real Estate	1.00%	20.00%	0.20%	2.00%	20.00%	0.40%	0.00%	-0.20%	0.00%	-0.20%
Commodities	-12.00%	4.00%	-0.48%	-10.00%	4.00%	-0.40%	0.00%	-0.08%	0.00%	-0.08%
Private Equity	2.00%	1.00%	0.02%	1.00%	1.00%	0.01%	0.00%	0.01%	0.00%	0.01%
Hedge Funds	1.00%	1.00%	0.01%	3.00%	1.00%	0.03%	0.00%	-0.02%	0.00%	-0.02%
Total	0.83%	100.00%	0.83%	1.59%	100.00%	1.59%	0.00%	-0.76%	0.00%	-0.76%

Remark: Here portfolio weights equals benchmark weights.

Total picture: example for single period

	Portfolio			Benchmark			Management Effects			Total
	Performance	Weight	Contribution	Performance	Weight	Contribution	Asset allocation	Stock picking	Interaction	
Cash	0.50%	10.00%	0.05%	0.10%	6.00%	0.01%	0.00%	0.02%	0.02%	0.04%
Domestic Bonds	1.00%	22.00%	0.22%	3.00%	20.00%	0.60%	0.06%	-0.40%	-0.04%	-0.38%
Foreign Bonds	3.00%	10.00%	0.30%	2.00%	15.00%	0.30%	-0.10%	0.15%	-0.05%	0.00%
Domestic Equities	-0.50%	17.00%	-0.09%	-0.10%	14.00%	-0.01%	0.00%	-0.06%	-0.01%	-0.07%
Foreign Equities	2.00%	12.00%	0.24%	4.00%	16.00%	0.64%	-0.16%	-0.32%	0.08%	-0.40%
Mortgages	5.00%	5.00%	0.25%	0.50%	3.00%	0.02%	0.01%	0.14%	0.09%	0.24%
Real Estate	1.00%	16.00%	0.16%	2.00%	20.00%	0.40%	-0.08%	-0.20%	0.04%	-0.24%
Commodities	-12.00%	4.00%	-0.48%	-10.00%	4.00%	-0.40%	0.00%	-0.08%	0.00%	-0.08%
Private Equity	2.00%	3.00%	0.06%	1.00%	1.00%	0.01%	0.02%	0.01%	0.02%	0.05%
Hedge Funds	1.00%	1.00%	0.01%	3.00%	1.00%	0.03%	0.00%	-0.02%	0.00%	-0.02%
Total	0.73%	100.00%	0.73%	1.59%	100.00%	1.59%	-0.25%	-0.76%	0.14%	-0.86%

Remark: Management effects should reflect the investment process.

Interaction effect - does it make sense?

- Interaction effect is not a "rest" term which can not be assigned to a specific management decision.
- Interaction effect is the result of the "interaction" of more than one decision.
- In case of a hierarchical investment process the interaction effect may be assigned to a specific management decision.
- Depending on the analytics software the interaction effect may be assigned to a specific management decision or smoothed by default - but this may lead to miss-interpretations.

Remark: Handling of interaction effect should reflect the investment process.

Different types of single factor based return attribution

Single factor based return attribution methods differ with respect to the way the excess return is decomposed, for example:

- Brinson and Fachler => opportunity costs considered if calculating the asset allocation effect.
- Brinson, Hood and Beebower => opportunity cost of 0% considered if calculating the asset allocation effect.
- Karnosky and Singer => introducing currency selection and risk premium.
- etc.

Remark: Return attribution methodology should reflect the investment process.

Equity versus fixed income return attribution

- "Equity return attribution" is mainly done for equity and multi asset class portfolios using a single factor based return attribution method because here the explanatory accuracy of multi factor based return attribution methods are limited, the investment process is more hierarchical and the individual steps of the investment process are more independent from each other.
- "Fixed income return attribution" is mainly done for fixed income portfolios using a multi factor based return attribution method because of the high explanatory accuracy and because the dependencies between the different factors may be very relevant for the findings of the performance analysis.

Remark: "Equity return attribution" is also applicable to fixed income portfolios but it needs a different interpretation.

The issue with multi period analysis

(1/2)

- If considering more than one reporting period one has to aggregate the different return contributions of the different periods to get the return contributions for the total period.
- The issue is that the addition of the return contributions of the different sub-periods do not add up with the return contribution for the total period because the compounding effect (inter-temporary cross-product) would be missing.
- There is no common way of compounding the management effects of the sub-periods and therefore different analytics softwares might come up with different results.

Remark: If using geometrical return contribution / attribution there is no issue with the inter-temporary cross-product.

The issue with multi period analysis

(2/2)

Period	Portfolio	Benchmark	Excess
1	5.00%	3.00%	2.00%
2	5.00%	3.00%	2.00%
3	5.00%	3.00%	2.00%
4	5.00%	3.00%	2.00%
5	5.00%	3.00%	2.00%
6	5.00%	3.00%	2.00%
7	5.00%	3.00%	2.00%
8	5.00%	3.00%	2.00%
9	5.00%	3.00%	2.00%
Compounding			5.15%
Total	55.13%	30.48%	24.66%

Different types of linking or smoothing algorithms

There is a big debate in the industry on how to link the sub-period contributions or on how to smooth the inter-temporary cross-product. Below are examples for currently used algorithms:

- by David R. Cariño
- by Jose G. Menchero
- by Andrew S. B. Frongello
- by GRAP (Groupe de Recherche en Attribution de Performance)
- by Bruce J. Feibel
- etc.

Remark: Linking or smoothing algorithms have no link to the decision making process and therefore no economical interpretation.

Contribution to excess return: multi period

(1/2)

Multi-period contribution =

[prior cumulated contribution x (1 + benchmark return current period)]
+ [current period contribution x (1 + prior cumulated portfolio return)]

=> Reference for the linking approach: "Investment Performance Measurement"; by Bruce J. Feibel

$$\begin{aligned} \text{ERM}_T &= \text{ERM}_{T-1} * (1 + R_T^{\text{BM}}) + \text{ER}_T * (1 + \text{RM}_{T-1}^{\text{P}}) \\ \text{ERM}_T &= \text{AAEM}_T^{\text{Total}} + \text{SPEM}_T^{\text{Total}} + \text{IAEM}_T^{\text{Total}} \\ \text{AAEM}_T^{\text{Total}} &= \text{AAEM}_{T-1}^{\text{Total}} * (1 + R_T^{\text{BM}}) + \text{AAE}_T^{\text{Total}} * (1 + \text{RM}_{T-1}^{\text{P}}) \\ \text{SPEM}_T^{\text{Total}} &= \text{SPEM}_{T-1}^{\text{Total}} * (1 + R_T^{\text{BM}}) + \text{SPE}_T^{\text{Total}} * (1 + \text{RM}_{T-1}^{\text{P}}) \\ \text{IAEM}_T^{\text{Total}} &= \text{IAEM}_{T-1}^{\text{Total}} * (1 + R_T^{\text{BM}}) + \text{IAE}_T^{\text{Total}} * (1 + \text{RM}_{T-1}^{\text{P}}) \end{aligned}$$

Contribution to excess return: multi period

(2/2)

ER_T	=	Excess return for subperiod T
ERM_T	=	Cumulated excess return until T
AAE_T^{Total}	=	Total asset allocation effect for subperiod T
$AAEM_T^{Total}$	=	Cumulated asset allocation effect until T
SPE_T^{Total}	=	Total stock picking effect for subperiod T
$SPEM_T^{Total}$	=	Cumulated total stock picking effect until T
IAE_T^{Total}	=	Total interaction effect for subperiod T
$IAEM_T^{Total}$	=	Cumulated total interaction effect until T
RM_T^P	=	Cumulated portfolio return until T
R_T^{BM}	=	Benchmark return for subperiod T

Total picture: second example for single period

	Portfolio			Benchmark			Management Effects			Total
	Performance	Weight	Contribution	Performance	Weight	Contribution	Asset allocation	Stock picking	Interaction	
Liquidität	0.50%	10.00%	0.05%	0.10%	6.00%	0.01%	0.00%	0.02%	0.02%	0.04%
Obligationen CHF	1.50%	22.00%	0.33%	0.50%	20.00%	0.10%	0.01%	0.20%	0.02%	0.23%
Obligationen Fremdwährungen	3.00%	10.00%	0.30%	4.50%	15.00%	0.68%	-0.23%	-0.23%	0.08%	-0.38%
Aktien Schweiz	2.00%	17.00%	0.34%	2.50%	14.00%	0.35%	0.08%	-0.07%	-0.02%	-0.01%
Aktien Ausland	0.50%	12.00%	0.06%	-2.00%	16.00%	-0.32%	0.08%	0.40%	-0.10%	0.38%
Hypotheken	0.50%	5.00%	0.03%	0.50%	3.00%	0.02%	0.01%	0.00%	0.00%	0.01%
Immobilien	2.00%	16.00%	0.32%	1.50%	20.00%	0.30%	-0.06%	0.10%	-0.02%	0.02%
Rohstoffe	6.00%	4.00%	0.24%	3.00%	4.00%	0.12%	0.00%	0.12%	0.00%	0.12%
Private Equity	2.00%	3.00%	0.06%	0.50%	1.00%	0.01%	0.01%	0.02%	0.03%	0.06%
Hedge Funds	1.00%	1.00%	0.01%	-1.50%	1.00%	-0.02%	0.00%	0.03%	0.00%	0.03%
Total	1.74%	100.00%	1.74%	1.24%	100.00%	1.24%	-0.10%	0.59%	0.01%	0.50%

Remark: Management effects should reflect the investment process.

Starting point: example for multi period

	Portfolio			Benchmark			Management Effects			Total
	Performance	Weight	Contribution	Performance	Weight	Contribution	Asset allocation	Stock picking	Interaction	
Cash	0.20%	6.00%	0.01%	0.20%	6.00%	0.01%	0.00%	0.00%	0.00%	0.00%
Domestic Bonds	3.52%	20.00%	0.71%	3.52%	20.00%	0.71%	0.00%	0.00%	0.00%	0.00%
Foreign Bonds	6.59%	15.00%	0.98%	6.59%	15.00%	0.98%	0.00%	0.00%	0.00%	0.00%
Domestic Equities	2.40%	14.00%	0.34%	2.40%	14.00%	0.34%	0.00%	0.00%	0.00%	0.00%
Foreign Equities	1.92%	16.00%	0.33%	1.92%	16.00%	0.33%	0.00%	0.00%	0.00%	0.00%
Mortgages	1.00%	3.00%	0.03%	1.00%	3.00%	0.03%	0.00%	0.00%	0.00%	0.00%
Real Estate	3.53%	20.00%	0.70%	3.53%	20.00%	0.70%	0.00%	0.00%	0.00%	0.00%
Commodities	-7.30%	4.00%	-0.28%	-7.30%	4.00%	-0.28%	0.00%	0.00%	0.00%	0.00%
Private Equity	1.51%	1.00%	0.02%	1.51%	1.00%	0.02%	0.00%	0.00%	0.00%	0.00%
Hedge Funds	1.46%	1.00%	0.02%	1.46%	1.00%	0.02%	0.00%	0.00%	0.00%	0.00%
Total	2.84%	100.00%	2.84%	2.84%	100.00%	2.84%	0.00%	0.00%	0.00%	0.00%

Remark: Here no bets, everything is indexed.

Asset allocation effect: example for multi period

	Portfolio			Benchmark			Management Effects			Total
	Performance	Weight	Contribution	Performance	Weight	Contribution	Asset allocation	Stock picking	Interaction	
Cash	0.20%	10.00%	0.02%	0.20%	6.00%	0.01%	0.01%	0.00%	0.00%	0.01%
Domestic Bonds	3.52%	22.00%	0.78%	3.52%	20.00%	0.71%	0.07%	0.00%	0.00%	0.07%
Foreign Bonds	6.59%	10.00%	0.65%	6.59%	15.00%	0.98%	-0.33%	0.00%	0.00%	-0.33%
Domestic Equities	2.40%	17.00%	0.41%	2.40%	14.00%	0.34%	0.07%	0.00%	0.00%	0.07%
Foreign Equities	1.92%	12.00%	0.25%	1.92%	16.00%	0.33%	-0.08%	0.00%	0.00%	-0.08%
Mortgages	1.00%	5.00%	0.05%	1.00%	3.00%	0.03%	0.02%	0.00%	0.00%	0.02%
Real Estate	3.53%	16.00%	0.56%	3.53%	20.00%	0.70%	-0.14%	0.00%	0.00%	-0.14%
Commodities	-7.30%	4.00%	-0.28%	-7.30%	4.00%	-0.28%	0.00%	0.00%	0.00%	0.00%
Private Equity	1.51%	3.00%	0.05%	1.51%	1.00%	0.02%	0.03%	0.00%	0.00%	0.03%
Hedge Funds	1.46%	1.00%	0.02%	1.46%	1.00%	0.02%	0.00%	0.00%	0.00%	0.00%
Total	2.49%	100.00%	2.49%	2.84%	100.00%	2.84%	-0.35%	0.00%	0.00%	-0.35%

Remark: Here portfolio returns equals the benchmark return.

Stock picking effect: example for multi period

	Portfolio			Benchmark			Management Effects			Total
	Performance	Weight	Contribution	Performance	Weight	Contribution	Asset allocation	Stock picking	Interaction	
Cash	1.00%	6.00%	0.06%	0.20%	6.00%	0.01%	0.00%	0.05%	0.00%	0.05%
Domestic Bonds	2.52%	20.00%	0.50%	3.52%	20.00%	0.71%	0.00%	-0.20%	0.00%	-0.20%
Foreign Bonds	6.09%	15.00%	0.91%	6.59%	15.00%	0.98%	0.00%	-0.08%	0.00%	-0.08%
Domestic Equities	1.49%	14.00%	0.21%	2.40%	14.00%	0.34%	0.00%	-0.13%	0.00%	-0.13%
Foreign Equities	2.51%	16.00%	0.41%	1.92%	16.00%	0.33%	0.00%	0.08%	0.00%	0.08%
Mortgages	5.53%	3.00%	0.17%	1.00%	3.00%	0.03%	0.00%	0.14%	0.00%	0.14%
Real Estate	3.02%	20.00%	0.60%	3.53%	20.00%	0.70%	0.00%	-0.10%	0.00%	-0.10%
Commodities	-6.72%	4.00%	-0.25%	-7.30%	4.00%	-0.28%	0.00%	0.04%	0.00%	0.04%
Private Equity	4.04%	1.00%	0.04%	1.51%	1.00%	0.02%	0.00%	0.03%	0.00%	0.03%
Hedge Funds	2.01%	1.00%	0.02%	1.46%	1.00%	0.02%	0.00%	0.00%	0.00%	0.00%
Total	2.67%	100.00%	2.67%	2.84%	100.00%	2.84%	0.00%	-0.17%	0.00%	-0.17%

Remark: Here portfolio weights equals benchmark weights.

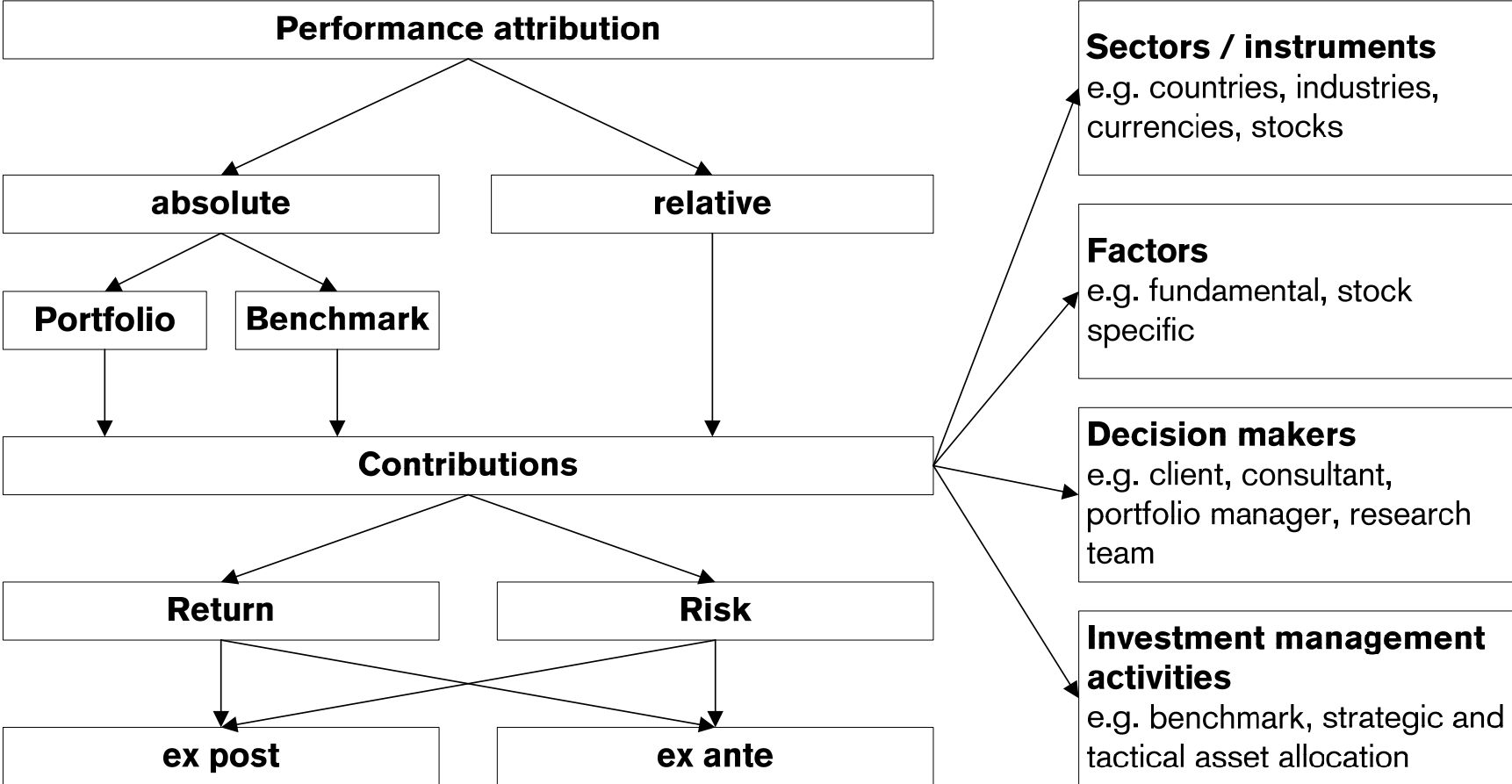
Total picture: example for multi period

	Portfolio			Benchmark			Management Effects			Total
	Performance	Weight	Contribution	Performance	Weight	Contribution	Asset allocation	Stock picking	Interaction	
Liquidität	1.00%	10.00%	0.10%	0.20%	6.00%	0.01%	0.01%	0.05%	0.03%	0.09%
Obligationen CHF	2.52%	22.00%	0.55%	3.52%	20.00%	0.71%	0.07%	-0.20%	-0.02%	-0.15%
Obligationen Fremdwährungen	6.09%	10.00%	0.61%	6.59%	15.00%	0.98%	-0.33%	-0.07%	0.02%	-0.38%
Aktien Schweiz	1.49%	17.00%	0.25%	2.40%	14.00%	0.34%	0.07%	-0.13%	-0.03%	-0.08%
Aktien Ausland	2.51%	12.00%	0.30%	1.92%	16.00%	0.33%	-0.08%	0.08%	-0.02%	-0.02%
Hypotheken	5.53%	5.00%	0.28%	1.00%	3.00%	0.03%	0.02%	0.14%	0.09%	0.25%
Immobilien	3.02%	16.00%	0.48%	3.53%	20.00%	0.70%	-0.14%	-0.10%	0.02%	-0.22%
Rohstoffe	-6.72%	4.00%	-0.25%	-7.30%	4.00%	-0.28%	0.00%	0.04%	0.00%	0.04%
Private Equity	4.04%	3.00%	0.12%	1.51%	1.00%	0.02%	0.03%	0.03%	0.05%	0.11%
Hedge Funds	2.01%	1.00%	0.02%	1.46%	1.00%	0.02%	0.00%	0.00%	0.00%	0.00%
Total	2.47%	100.00%	2.47%	2.84%	100.00%	2.84%	-0.35%	-0.17%	0.15%	-0.37%

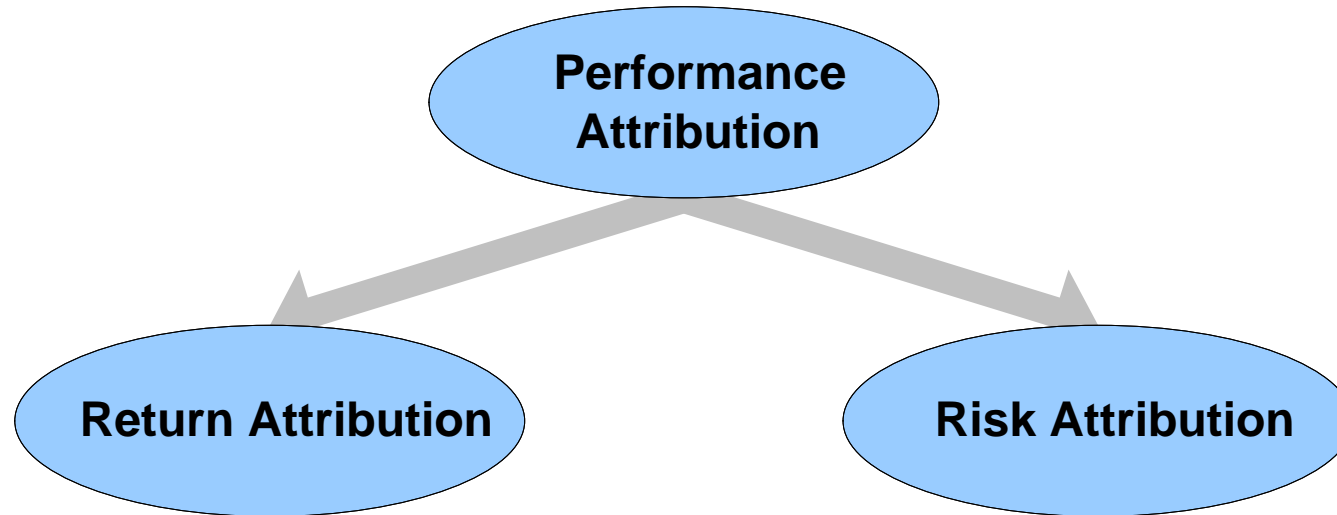
Remark: Management effects should reflect the investment process.

2. Introduction to risk attribution

Performance attribution: the big picture



Risk attribution as part of performance attribution



Groups of FT Sector	Asset Allocation	Stock Selection	Interaction	Total
Basic Industries	-0.07%	-0.04%	0.01%	0.04%
Cyclical Consumer Goods	-0.21%	0.26%	-0.11%	-0.06%
Cyclical Services	-0.05%	0.00%	-0.09%	-0.13%
Finance	0.00%	0.11%	0.00%	0.11%
General Industries	0.10%	-0.19%	-0.14%	-0.22%
Information Tech	0.23%	-0.36%	0.08%	-0.07%
Non Cyclical Cons Goods	-0.28%	-0.15%	0.01%	-0.41%
Non Cyclical Services	-0.38%	-0.13%	0.00%	-0.51%
Resources	-0.22%	-0.02%	0.00%	-0.24%
Utilities	-0.13%	0.00%	-0.01%	-0.14%
Other Assets	0.08%	0.00%	0.01%	0.09%
Total	-0.73%	-0.52%	-0.25%	-1.56%

Risk Model : Global	Portfolio	Benchmark
Number of Securities	99	576
Number of Currencies	5	0
Portfolio Value	227'447'728	
Total Risk (ex-ante)	15.76%	15.31%
- Factor Specific Risk	15.53%	15.20%
- Stock Specific Risk	2.72%	1.83%
Tracking Error (ex-ante)	2.35%	
Relative Value at Risk	10'878'425	
R-squared	0.98	
Beta-adjusted Risk	15.59%	15.31%
Predicted Beta	1.02	
Predicted Dividend Yield	2.22	2.37
P/E Ratio (E: 12 months)	38.19	29.42
P/B Ratio (B: year-end)	5.34	4.68

Risk Model : Global	Portfolio	Tracking Error
Total Risk (ex-ante)	15.76%	2.35%
Factor Specific Risk	15.53%	1.40%
- Region	12.40%	0.19%
- Country	9.06%	1.13%
- Industry	3.07%	0.72%
- Fundamental	1.10%	0.48%
- Currency	4.36%	0.52%
- Covariance (+/-)	4.93%	0.60%
Stock Specific Risk	2.72%	1.89%

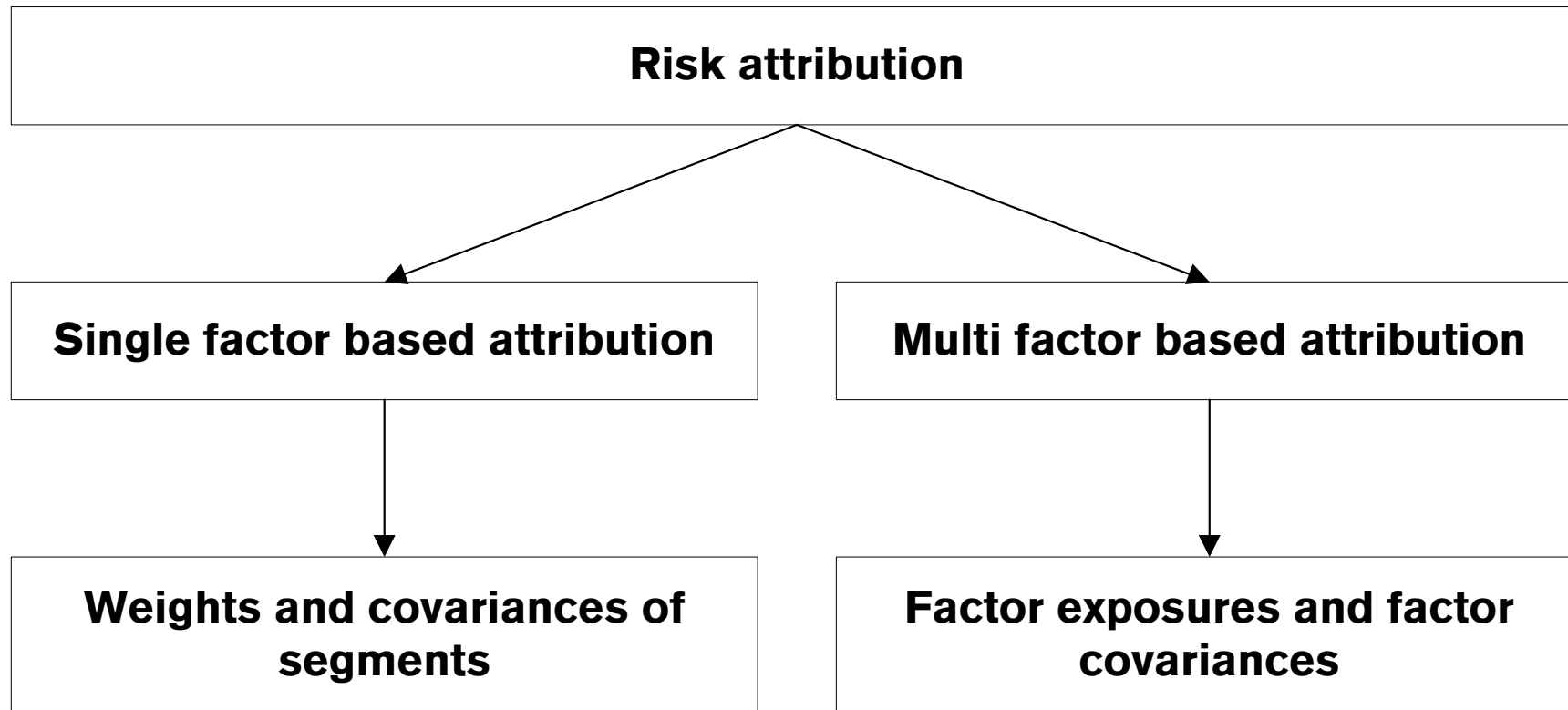
Definition of risk attribution

- Risk attribution is the measurement and quantification of the historical as well as expected risk contributions of the individual steps of the investment process as well as of the applied financial instruments.
- We distinguish between risk contribution and attribution, whereby risk contribution is a more or less arbitrary breakdown of the risk using a given breakdown of the investment universe and risk attribution is a decision oriented decomposition of the risk.
- We distinguish between single factor based attribution and multi factor based attribution where the former approach is mainly used for multi asset class portfolios and the latter approach is used mainly for equity and fixed income portfolios.

Purpose of risk attribution

1. Identification of risks.
2. Measurement of risk and sources of risk.
3. Explain on what decisions the risk and the sources of risk are based on.
4. Support of portfolio management decisions.
5. Monitoring the implementation of the investment strategy.
6. Objectification of the discussions on risk.

Risk attribution: the big picture



Remark: In the following we focus on single factor based attribution.

Risk attribution: necessary inputs

- Weights of the different asset classes or asset segments for the portfolio for the **relevant** period.
- Weights of the different asset classes or asset segments for the benchmark for the **relevant** period.
- Estimated covariances between asset classes or asset segments for the portfolio and the benchmark for the **relevant** period.

Some basics

(1/3)

- The absolute risk of a portfolio is often estimated, measured or expressed using the statistical measure variance (or volatility).

$$\sigma^2(R_i) = E[(R_i - E(R_i))^2] \text{ or } \sigma(R_i) = \sqrt{E(R_i - E(R_i))^2}$$

- Variance is easy to interpret, is measured in units of return and is easy to estimate.
- Often it is assumed that returns are lognormal distributed and therefore that the variance can be best estimated using the unbiased estimator.

$$\sigma^2(R_i) = \frac{1}{n-1} * \sum_{i=1}^n (R_i - \bar{R})^2 \text{ or } \sigma(R_i) = \sqrt{\frac{1}{n-1} * \sum_{i=1}^n (R_i - \bar{R})^2}$$

Some basics

(2/3)

- Since covariances are additive the risk of a portfolio can easily be decomposed.

$$\begin{aligned}R^P &= \sum_{i=1}^n w_i^P * R_i^P \Rightarrow \\ \sigma^2(R^P) &= \sum_{i=1}^n \sum_{j=1}^n w_i^P * w_j^P * \text{Cov}(R_i^P, R_j^P) \\ \sigma^2(R^P) &= \sum_{i=1}^n (w_i^P)^2 * \sigma^2(R_i^P) + \sum_{\substack{i=1 \\ i \neq j}}^n \sum_{j=1}^n w_i^P * w_j^P * \text{Cov}(R_i^P, R_j^P) \\ \sigma^2(R^P) &= \sum_{i=1}^n (w_i^P)^2 * \sigma^2(R_i^P) + \sum_{\substack{i=1 \\ i \neq j}}^n \sum_{j=1}^n w_i^P * w_j^P * \text{Correl}(R_i^P, R_j^P) * \sigma(R_i^P) * \sigma(R_j^P)\end{aligned}$$

- For risk attribution only variances can be used and if returns are not uncorrelated a lot of covariances have to be decomposed which can not be assigned clearly.

Some basics

(3/3)

- Relative risk is often estimated, measured or expressed using the statistical measure tracking error, i.e. the variance (volatility) of the excess returns of the portfolio versus the benchmark, and can easily be decomposed.

$$\begin{aligned} ER &= R^P - R^{BM} \Rightarrow \\ \sigma^2(ER) &= \sigma^2(R^P - R^{BM}) \\ \sigma^2(ER) &= \sigma^2(R^P) + \sigma^2(R^{BM}) - 2 * Cov(R^P, R^{BM}) \end{aligned}$$

- Risk models often are designed using the CAPM- and APT-theory, of course other types of risk models are also used.

CAPM risk model

- CAPM based risk model (historical securities weights and securities returns).

$$R^P = \sum_{i=1}^n w_i^P * R_i^P \Rightarrow$$

$$\sigma^2(R^P) = \sum_{i=1}^n (w_i^P)^2 * \sigma^2(R_i^P) + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n w_i^P * w_j^P * \text{Cov}(R_i^P, R_j^P)$$

$$\sigma^2(R^P) = \begin{pmatrix} w_1^P \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ w_n^P \end{pmatrix}^T * \begin{bmatrix} \sigma^2(R_1^P) & \cdot & \cdot & \cdot & \cdot & \text{Cov}(R_1^P, R_n^P) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \text{Cov}(R_n^P, R_1^P) & \cdot & \cdot & \cdot & \cdot & \sigma^2(R_n^P) \end{bmatrix} * \begin{pmatrix} w_1^P \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ w_n^P \end{pmatrix}$$

Risk attribution using CAPM

- Decomposing the absolute risk for predefined segments "k", e.g. assets, asset classes, sectors, styles, duration buckets, etc.

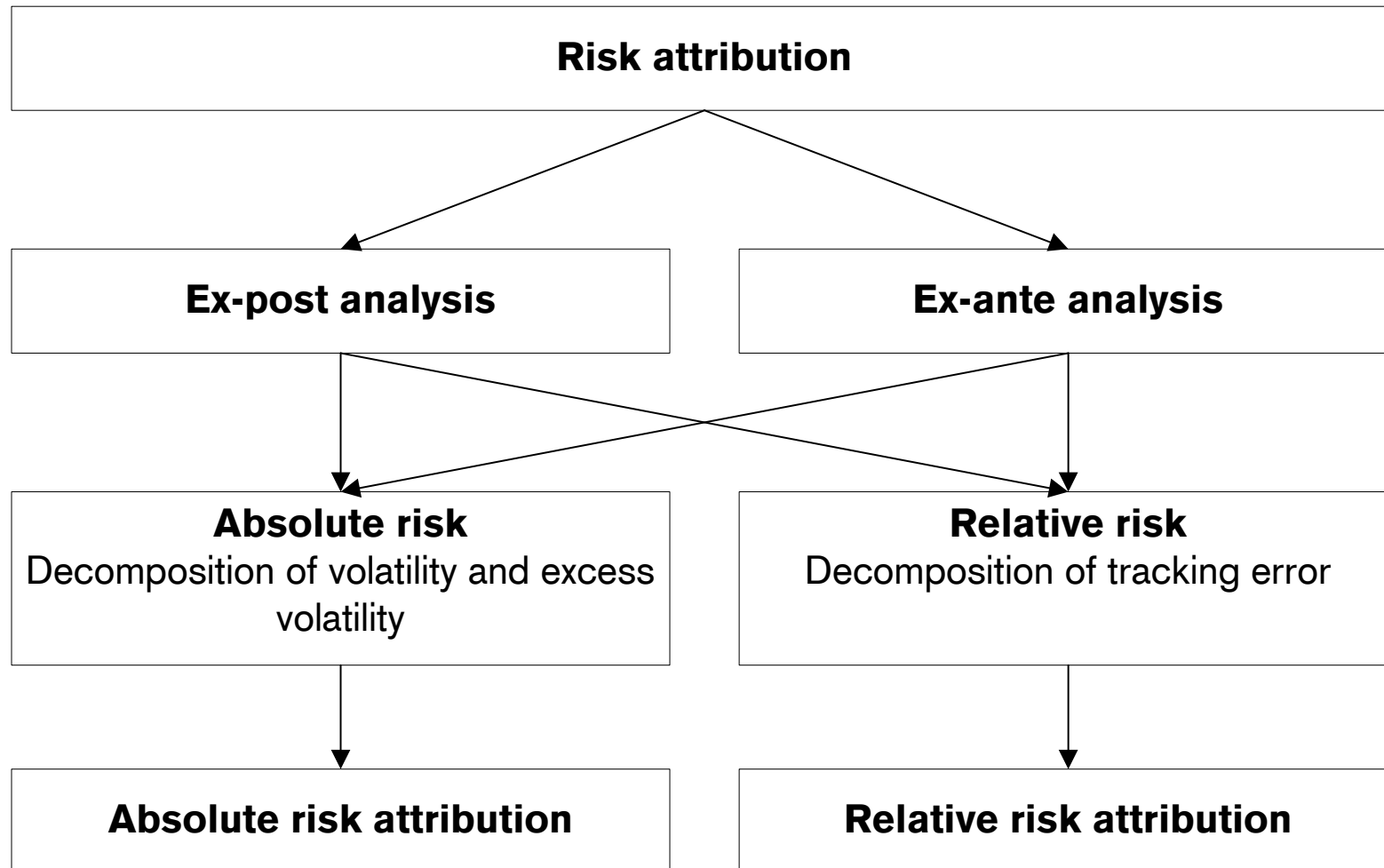
$$R^P = \sum_{k=1}^m w_k^P * R_k^P \Rightarrow$$
$$\sigma^2(R^P) = \sum_{k=1}^m (w_k^P)^2 * \sigma^2(R_k^P) + \sum_{k=1}^m \sum_{\substack{n=1 \\ n \neq k}}^m w_k^P * w_n^P * \text{Cov}(R_k^P, R_n^P)$$

- Decomposing the relative risk follows the same procedure as described before.
=> How does factor "k" contribute to the absolute or relative risk?

Discussion of risk attribution using CAPM

- Estimation of lots of covariances.
- Availability of long time series for all securities.
- Integration of new securities is difficult.
- "Slice and dice" decomposition is not possible, i.e. risk attribution has to be redesigned if changing perspective.
- "Look through" risk decomposition is not possible.
- Fraction of the non-assignable covariances can be substantial.
- Random distributed covariances have normally no economical value.
- Neglect of important risk factors, analysis is often data driven.
- Simple risk model which is normally easy to estimate.

Types of risk attribution



Ex-ante risk attribution: definitions

- **Ex-ante absolute risk**
 - is measured using the variance
- **Ex-ante excess risk**
 - is measured using the variance (variance tracking error)
- **Ex-ante absolute excess risk**
 - difference between the absolute risk of two portfolios
- **Ex-ante relative excess risk**
 - Variance (variance tracking error) of the return differences of two portfolios

Remark: Also other risk figures like VaR can be decomposed.

Ex-ante absolute risk attribution: general idea

- **Step 1:** Calculate the absolute risk for the portfolio and for the benchmark and analyze the individual figures and their difference.
- **Step 2:** Furthermore decompose the figures according to management activities or/and asset classes.

$$\begin{bmatrix} w_1^P & \dots & w_n^P \end{bmatrix} \begin{bmatrix} \text{Cov}_E^P \end{bmatrix} \begin{bmatrix} w_1^P \\ \vdots \\ w_n^P \end{bmatrix} - \begin{bmatrix} w_1^{BM} & \dots & w_n^{BM} \end{bmatrix} \begin{bmatrix} \text{Cov}_E^{BM} \end{bmatrix} \begin{bmatrix} w_1^{BM} \\ \vdots \\ w_n^{BM} \end{bmatrix}$$

Cov_E^P = Expected covariance matrix for portfolio

Cov_E^{BM} = Expected covariance matrix for benchmark

Ex-ante absolute risk attribution using the Brinson, Hood, Beebower method (1/4)

- Absolute excess risk = absolute risk of portfolio
- absolute risk of benchmark

$$Ri_{AE} = Ri_{AE}^P - Ri_{AE}^{BM}$$
$$= \sum_{i=1}^n \sum_{j=1}^n w_i^P * w_j^P * Cov_E^P(i,j) - \sum_{i=1}^n \sum_{j=1}^n w_i^{BM} * w_j^{BM} * Cov_E^{BM}(i,j)$$

- Absolute excess risk = risk contribution due to asset allocation
+ risk contribution due to stock picking
+ risk contribution due to interaction

$$Ri_{AE} = Ri_{AE}^{AA} + Ri_{AE}^{SP} + Ri_{AE}^{IA}$$

Ex-ante absolute risk attribution using the Brinson, Hood, Beebower method (2/4)

- Absolute excess risk contribution due to asset allocation

$$Ri_{AE}^{AA} = \sum_{i=1}^n \sum_{j=1}^n [(w_i^P * w_j^P) - (w_i^{BM} * w_j^{BM})] * Cov_E^{BM}(i, j)$$

- Absolute excess risk contribution due to stock picking

$$Ri_{AE}^{SP} = \sum_{i=1}^n \sum_{j=1}^n w_i^{BM} * w_j^{BM} * [Cov_E^P(i, j) - Cov_E^{BM}(i, j)]$$

- Absolute excess risk contribution due to interaction

$$Ri_{AE}^{IA} = \sum_{i=1}^n \sum_{j=1}^n [(w_i^P * w_j^P) - (w_i^{BM} * w_j^{BM})] * [Cov_E^P(i, j) - Cov_E^{BM}(i, j)]$$

Ex-ante absolute risk attribution using the Brinson, Hood, Beebower method (3/4)

- Absolute excess risk contribution due to asset allocation for asset class i

$$Ri_{AE}^{AA}(i) = \sum_{j=1}^n [(w_i^P * w_j^P) - (w_i^{BM} * w_j^{BM})] * Cov_E^{BM}(i, j)$$

- Absolute excess risk contribution due to stock picking for asset class i

$$Ri_{AE}^{SP}(i) = \sum_{j=1}^n w_i^{BM} * w_j^{BM} * [Cov_E^P(i, j) - Cov_E^{BM}(i, j)]$$

Ex-ante absolute risk attribution using the Brinson, Hood, Beebower method (4/4)

- Absolute excess risk contribution due to interaction for asset class i

$$Ri_{AE}^{IA}(i) = \sum_{j=1}^n [(w_i^P * w_j^P) - (w_i^{BM} * w_j^{BM})] * [Cov_E^P(i, j) - Cov_E^{BM}(i, j)]$$

=> assuming distributed covariances by rows

Ex-ante relative risk attribution: general idea

- **Step 1:** Generate a long/short portfolio where the long position consists of the portfolio and the short position is the corresponding benchmark.
- **Step 2:** Calculate and analyse the absolute risk for this long/short portfolio.
- **Step 3:** Furthermore decompose the figures according to management activities or/and asset classes.

$$\begin{bmatrix} w_1^P & \dots & w_n^P & -w_1^{BM} & \dots & -w_n^{BM} \end{bmatrix} \begin{bmatrix} \left[\begin{array}{c} \text{Cov}_E^P \\ \text{Cov}_E^{P,BM} \end{array} \right] \\ \left[\begin{array}{c} \text{Cov}_E^{P,BM} \\ \text{Cov}_E^{BM} \end{array} \right] \end{bmatrix} \begin{bmatrix} w_1^P \\ \vdots \\ w_n^P \\ -w_1^{BM} \\ \vdots \\ -w_n^{BM} \end{bmatrix}$$

Ex-ante relative risk attribution using the Brinson, Hood, Beebower method (1/5)

- Relative risk = tracking error of the portfolio versus the benchmark
 - = absolute risk of portfolio
 - + absolute risk of benchmark
 - diversification effect portfolio versus benchmark

$$\begin{aligned}
 Ri_{RE} &= TE_E(P;B) \\
 &= Ri_{AE}^P + Ri_{AE}^{BM} - Di_E(P;B) \\
 &= \sum_{i=1}^n \sum_{j=1}^n w_i^P * w_j^P * Cov_E^P(i,j) + \sum_{i=1}^n \sum_{j=1}^n w_i^{BM} * w_j^{BM} * Cov_E^{BM}(i,j) - 2 * \sum_{i=1}^n \sum_{j=1}^n w_i^P * w_j^{BM} * Cov_E^{P,BM}(i,j)
 \end{aligned}$$

or in matrix notation

$$= \begin{bmatrix} w_i^P & -w_i^{BM} \end{bmatrix} * \begin{bmatrix} Cov_E^P & Cov_E^{P,BM} \\ Cov_E^{P,BM} & Cov_E^{BM} \end{bmatrix} * \begin{bmatrix} w_i^P \\ -w_i^{BM} \end{bmatrix} \quad (2n \times 2n \text{ matrix}); \text{ for } i = 1, \dots, n$$

Ex-ante relative risk attribution using the Brinson, Hood, Beebower method (2/5)

- Relative risk = relative risk contribution due to asset allocation
+ relative risk contribution due to stock picking
+ relative risk contribution due to interaction

$$Ri_{RE} = Ri_{RE}^{AA} + Ri_{RE}^{SP} + Ri_{RE}^{IA}$$

Ex-ante relative risk attribution using the Brinson, Hood, Beebower method (3/5)

- Relative risk contribution due to asset allocation

$$\begin{aligned}
 Ri_{RE}^{AA} &= \sum_{i=1}^n \sum_{j=1}^n 0 * (w_i^P - w_i^{BM}) * (\text{Cov}_E^P(i, j) + \text{Cov}_E^{BM}(i, j) - 2 * \text{Cov}_E^{P, BM}(i, j)) \\
 &= \sum_{i=1}^n \sum_{j=1}^n 0 * 0 * \text{Cov}_E^P(i, j) + \sum_{i=1}^n \sum_{j=1}^n (w_i^P - w_i^{BM}) * (w_j^P - w_j^{BM}) * \text{Cov}_E^{BM}(i, j) \\
 &\quad - 2 * \sum_{i=1}^n \sum_{j=1}^n 0 * (w_i^P - w_i^{BM}) * \text{Cov}_E^{P, BM}(i, j) \\
 &= \sum_{i=1}^n \sum_{j=1}^n (w_i^P - w_i^{BM}) * (w_j^P - w_j^{BM}) * \text{Cov}_E^{BM}(i, j)
 \end{aligned}$$

or in matrix notation

$$\begin{aligned}
 &= \begin{bmatrix} 0 & w_i^P - w_i^{BM} \end{bmatrix} * \begin{bmatrix} \text{Cov}_E^P & \text{Cov}_E^{P, BM} \\ \text{Cov}_E^{P, BM} & \text{Cov}_E^{BM} \end{bmatrix} * \begin{bmatrix} 0 \\ w_i^P - w_i^{BM} \end{bmatrix} \quad (2n \times 2n \text{ matrix}); \text{ for } i = 1, \dots, n \\
 &= [w_i^P - w_i^{BM}] * [\text{Cov}_E^{BM}(i, j)] * [w_i^P - w_i^{BM}] \quad (n \times n \text{ matrix}); \text{ for } i = 1, \dots, n
 \end{aligned}$$

Ex-ante relative risk attribution using the Brinson, Hood, Beebower method (4/5)

- Relative risk contribution due to stock picking

$$\begin{aligned}
 Ri_{RE}^{SP} &= \sum_{i=1}^n \sum_{j=1}^n w_i^{BM} * w_j^{BM} * Cov_E^P(i, j) + \sum_{i=1}^n \sum_{j=1}^n w_i^{BM} * w_j^{BM} * Cov_E^{BM}(i, j) \\
 &\quad - 2 * \sum_{i=1}^n \sum_{j=1}^n w_i^{BM} * w_j^{BM} * Cov_E^{P,BM}(i, j) \\
 &= \sum_{i=1}^n \sum_{j=1}^n w_i^{BM} * w_j^{BM} * (Cov_E^P(i, j) + Cov_E^{BM}(i, j) - 2 * Cov_E^{P,BM}(i, j))
 \end{aligned}$$

or in matrix notation

$$= \begin{bmatrix} w_i^{BM} & -w_i^{BM} \end{bmatrix} * \begin{bmatrix} Cov_E^P & Cov_E^{P,BM} \\ Cov_E^{P,BM} & Cov_E^{BM} \end{bmatrix} * \begin{bmatrix} w_i^{BM} \\ -w_i^{BM} \end{bmatrix} \quad (2n \times 2n \text{ matrix}); \text{ for } i = 1, \dots, n$$

Ex-ante relative risk attribution using the Brinson, Hood, Beebower method (5/5)

- Relative risk contribution due to interaction

$$\begin{aligned}
 Ri_{RE}^{IA} &= Ri_{RE} - Ri_{RE}^{AA} - Ri_{RE}^{SP} \\
 &= \begin{bmatrix} w_i^P & -w_i^{BM} \end{bmatrix} * \begin{bmatrix} Cov_E^P & Cov_E^{P,BM} \\ Cov_E^{P,BM} & Cov_E^{BM} \end{bmatrix} * \begin{bmatrix} w_i^P \\ -w_i^{BM} \end{bmatrix} \\
 &\quad - \begin{bmatrix} 0 & w_i^P - w_i^{BM} \end{bmatrix} * \begin{bmatrix} Cov_E^P & Cov_E^{P,BM} \\ Cov_E^{P,BM} & Cov_E^{BM} \end{bmatrix} * \begin{bmatrix} 0 \\ w_i^P - w_i^{BM} \end{bmatrix} \\
 &\quad - \begin{bmatrix} w_i^{BM} & -w_i^{BM} \end{bmatrix} * \begin{bmatrix} Cov_E^P & Cov_E^{P,BM} \\ Cov_E^{P,BM} & Cov_E^{BM} \end{bmatrix} * \begin{bmatrix} w_i^{BM} \\ -w_i^{BM} \end{bmatrix} ; \text{ for } i = 1, \dots, n
 \end{aligned}$$

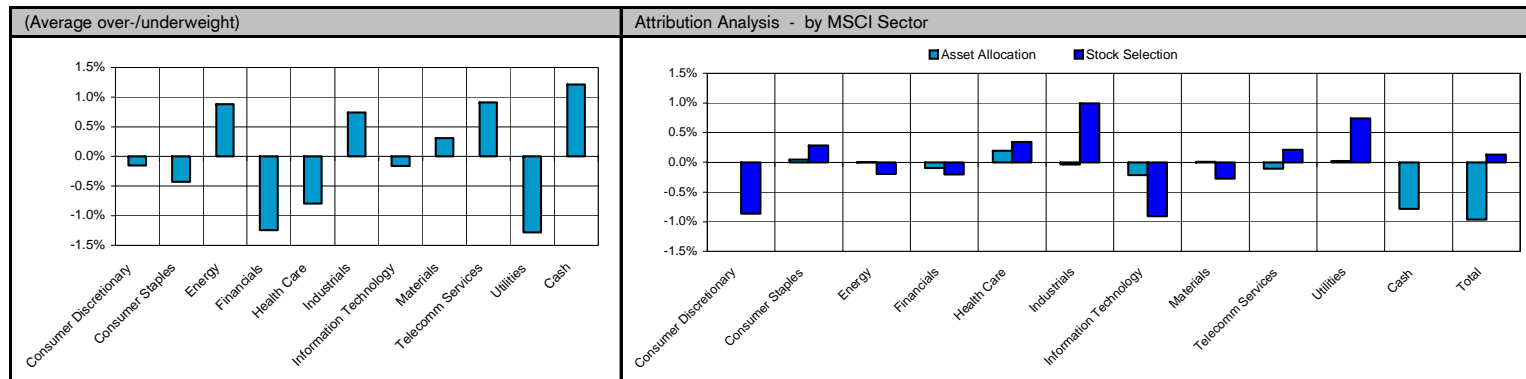
- Relative risk contribution due to management activities for asset class i by summing up all covariances for the row i

3. Example for an equity return and risk attribution

Example for an equity return and risk attribution

(1/6)

Portfolio		Return		Attribution Effects	Attribution by MSCI Sector	Attribution by 5 World Regions
NAME (ID)	Composite World MSCI, active, mandates	Currency	CHF	Asset Allocation	-0.96%	-1.01%
PM	AMPE	Return Portfolio	18.28%	Stock Selection	0.13%	-0.59%
BENCHMARK	MSCI World in CHF	Return Benchmark	19.60%	Interaction	-0.49%	0.28%
PERIOD	31.12.2002 - 31.12.2003	Return Relative	-1.32%	Total	-1.32%	-1.32%



Risk Analysis (end period)	Portfolio	Benchmark	Attribution Analysis - by 5 World Regions
Number of Securities	67	1'550	
Number of Currencies	8	0	
Portfolio Value	84'334'091		
Total Risk (ex-ante)	18.81%	18.21%	
- Factor Specific Risk	18.66%	18.18%	
- Stock Specific Risk	2.39%	1.02%	
Tracking Error (ex-ante)	2.57%		
Value at Risk (at 95%)	3'570'469		
R-squared	0.98		
Beta-adjusted Risk	18.64%	18.21%	
Predicted Beta	1.02		
Predicted Dividend Yield	1.86	2.01	
P/E Ratio (E: 12 months)	28.39	26.00	Important Remark: Differences between attribution returns and the returns of the official performance measurement tool are usual. They can be explained by the two systems using two different methodologies and by intraday trading gains or losses. Above figures are subject to future changes.
P/B Ratio (B: year-end)	2.58	2.56	

Example for an equity return and risk attribution

(2/6)

by MSCI Sector	Average PF Weight	Average BM Weight	Relative Weight	Absolute PF Return	Absolute BM Return	Difference PF-BM	Difference BM-BM Tot
Consumer Discretionary	12.03%	12.18%	-0.15%	15.58%	23.07%	-7.49%	3.47%
Consumer Staples	8.83%	9.26%	-0.43%	7.74%	5.09%	2.65%	-14.51%
Energy	8.29%	7.41%	0.88%	10.51%	13.30%	-2.79%	-6.30%
Financials	21.70%	22.94%	-1.24%	23.90%	24.94%	-1.04%	5.34%
Health Care	11.73%	12.53%	-0.80%	9.60%	7.31%	2.29%	-12.30%
Industrials	10.45%	9.71%	0.74%	34.93%	24.08%	10.85%	4.48%
Information Technology	12.33%	12.49%	-0.16%	25.06%	32.79%	-7.73%	13.19%
Materials	4.84%	4.53%	0.31%	23.11%	30.33%	-7.22%	10.73%
Telecomm Services	6.15%	5.24%	0.91%	15.02%	12.64%	2.38%	-6.96%
Utilities	2.44%	3.72%	-1.28%	34.09%	15.45%	18.64%	-4.15%
Cash	1.21%	0.00%	1.21%	8.94%	0.00%	8.94%	-19.60%
Total	100.00%	100.00%	0.00%	18.28%	19.60%	-1.32%	0.00%

Import remarks:

- The **weight numbers** gives an overview of the average weight invested in the different groups (e.g. sectors) with daily weights averaged over the chosen period.
- The **absolute return numbers** give an overview of the group's absolute performance (e.g. sector) within the chosen time period (portfolio and benchmark)
- The **difference PF-BM** compares the group's performance of the portfolio to the group's performance in the benchmark within the chosen time period.
- The **difference BM-BM Tot** compares the group's performance in the benchmark with the performance of the total benchmark within the chosen time period.

Please note that cash is included in the relevant country or regional groups and shown as "other assets" else (e.g. in sectors). Concerning derivatives, only futures are split up at the moment; derivatives (call or put options) are not yet included. Moreover, illiquid securities as private placements are not yet taken into account.

Example for an equity return and risk attribution

(3/6)

by MSCI Sector	Asset Allocation	Stock Selection	Interaction		Total
Consumer Discretionary	0.00%	-0.86%	-0.03%		-0.89%
Consumer Staples	0.05%	0.28%	0.03%		0.36%
Energy	0.00%	-0.19%	-0.02%		-0.21%
Financials	-0.10%	-0.21%	0.01%		-0.30%
Health Care	0.20%	0.35%	-0.03%		0.52%
Industrials	-0.04%	0.99%	0.12%		1.07%
Information Technology	-0.21%	-0.91%	0.01%		-1.11%
Materials	0.01%	-0.28%	-0.08%		-0.35%
Telecomm Services	-0.11%	0.21%	-0.08%		0.02%
Utilities	0.02%	0.74%	-0.41%		0.35%
Cash	-0.78%	0.00%	0.00%		-0.78%
Total	-0.96%	0.13%	-0.49%		-1.32%

Import remarks:

Asset Allocation Effect is the portion of portfolio excess return that is attributable to taking different group bets from the benchmark. An overweight of a group (e.g. SPI sector "Chemicals") that outperforms the whole benchmark (e.g. SPI) will generate a positive asset allocation effect.

Security Selection Effect is the portion of portfolio excess return attributable to choosing different securities within groups from the benchmark. An overweight of a well-performing security (e.g. Novartis) in comparison to its group benchmark (e.g. SPI sector "Chemicals") will generate a positive stock selection effect.

Interaction Effect is the portion of the portfolio excess return which is not attributable to asset allocation not stock selection.

Example for an equity return and risk attribution

(6/6)

Risk Model: Global	Portfolio	Benchmark
Number of Securities	67	1'550
Number of Currencies	8	0
Portfolio Value	84'334'091	
Total Risk (ex-ante)	18.81%	18.21%
- Factor Specific Risk	18.66%	18.18%
- Stock Specific Risk	2.39%	1.02%
Tracking Error (ex-ante)	2.57%	
Relative Value at Risk	3'570'469	
R-squared	0.98	
Beta-adjusted Risk	18.64%	18.21%
Predicted Beta	1.02	
Predicted Dividend Yield	1.86	2.01
P/E Ratio (E: 12 months)	28.39	26.00
P/B Ratio (B: year-end)	2.58	2.56

Risk Model: Global	Portfolio	Tracking Error
Total Risk (ex-ante)	18.81%	2.57%
Factor Specific Risk	18.66%	1.50%
- Region	11.50%	0.18%
- Country	6.98%	0.83%
- Industry	2.64%	0.77%
- Fundamental	1.44%	0.78%
- Currency	8.42%	0.27%
- Covariance (+/-)	9.35%	0.52%
Stock Specific Risk	2.39%	2.08%

Explication of risk model: Factor risk is a standard deviation that is measured by multiplying the 5-year exposure of the components of a portfolio to each risk factor and by multiplying these figures by the externally determined risk of each factor. The Tracking Error is measured similarly except that it is the *difference* between portfolio and benchmark exposure that is multiplied. Specific Risk is the standard deviation that measures the volatility of the risk not captured by the factor model. The model consists of 3 regional, 21 country, 38 industry and 8 fundamental factors (market cap, 4-year E/P growth, E/P, B/P, 5-year yield, long term debt, 5-year ROE variability and 5-year earnings variability).

4. Example for a fixed income return and risk attribution

Example for an fixed income return and risk attribution(1/6)

Name (ID) Sample Fixed Income
 Benchmark JP Morgan Global Traded
 Period 2008.01.31 to 2008.02.29
 Currency Europe - Euro

RETURN OVERVIEW	
Return PF	-0.25%
Return BM	-0.13%
Return Relative	-0.12%

RETURN ATTRIBUTION EFFECTS	
Yield	0.01%
Duration	0.01%
Term Structure	-0.12%
Sector	-0.25%
Quality	-0.02%
Other Spread	0.07%
Currency	-0.21%
Hedging	0.21%
Selection	0.18%

RISK ATTRIBUTION EFFECTS	
Risk Portfolio	5.04%
Risk Benchmark	5.28%
Tracking Error	0.49%

Example for an fixed income return and risk attribution(2/6)

ANALYSIS 1 - WEIGHTS AND DURATION (ENDING PERIOD)

Currency	PF Weight	BM Weight	Rel Weight		PF Duration	BM Duration	Rel Duration
Switzerland	---	---	---		---	---	---
Euro	28.62%	37.37%	-8.75%		1.94	2.32	-0.38
United Kingdom	10.98%	6.15%	4.83%		0.55	0.58	-0.03
Denmark	0.00%	0.71%	-0.71%		0.00	0.04	-0.04
Norway	0.00%	0.00%	0.00%		0.00	0.00	0.00
Sweden	2.23%	0.63%	1.60%		0.02	0.04	-0.02
Czech Republic	---	---	---		---	---	---
Hungary	---	---	---		---	---	---
Poland	---	---	---		---	---	---
United States	25.01%	21.56%	3.45%		1.25	1.15	0.10
Canada	1.37%	2.01%	-0.64%		0.09	0.14	-0.05
Japan	30.94%	31.22%	-0.28%		1.46	1.98	-0.52
Australia	0.61%	0.36%	0.25%		0.01	0.02	-0.01
Hong Kong	---	---	---		---	---	---
New Zealand	---	---	---		---	---	---
Singapore	---	---	---		---	---	---
South Korea	---	---	---		---	---	---
Mexico	---	---	---		---	---	---
South Africa	---	---	---		---	---	---
Others	---	---	---		---	---	---
Total	100 %	100 %	0%		5.32	6.27	-0.95

Example for an fixed income return and risk attribution(3/6)

ANALYSIS 2 - RETURN ATTRIBUTION EFFECTS (WHOLE PERIOD)

Currency	PF Return	BM Return	Yield	Duration	Term Structure	Sector	Quality	Other Spread	Currency	Hedging	Selection	TOTAL
Switzerland	---	---	---	---	---	---	---	---	---	---	---	---
Euro	0.12	0.20	-0.045	-0.163	0.082	-0.018	0.000	0.043	0.000	0.000	0.025	-0.076
United Kingdom	-0.32	-0.14	0.029	0.135	-0.284	-0.056	-0.013	0.000	-0.130	0.125	0.018	-0.183
Denmark	0.00	0.01	-0.002	-0.011	0.008	0.000	0.000	0.000	0.000	0.000	0.000	-0.006
Norway	0.00	0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sweden	0.04	0.01	0.008	0.004	-0.004	0.000	0.000	0.000	0.031	-0.008	-0.002	0.029
Czech Republic	---	---	---	---	---	---	---	---	---	---	---	---
Hungary	---	---	---	---	---	---	---	---	---	---	---	---
Poland	---	---	---	---	---	---	---	---	---	---	---	---
United States	-0.13	-0.30	0.013	0.053	0.130	-0.103	-0.007	0.024	-0.101	0.069	0.096	0.174
Canada	0.04	0.04	-0.002	-0.024	0.020	0.000	0.000	0.000	-0.004	0.000	0.000	-0.008
Japan	-0.01	0.04	0.010	0.010	-0.069	-0.073	0.000	0.000	-0.012	0.027	0.050	-0.052
Australia	0.02	0.01	0.002	0.001	0.001	0.000	0.000	0.000	0.005	0.000	0.000	0.009
Hong Kong	---	---	---	---	---	---	---	---	---	---	---	---
New Zealand	---	---	---	---	---	---	---	---	---	---	---	---
Singapore	---	---	---	---	---	---	---	---	---	---	---	---
South Korea	---	---	---	---	---	---	---	---	---	---	---	---
Mexico	---	---	---	---	---	---	---	---	---	---	---	---
South Africa	---	---	---	---	---	---	---	---	---	---	---	---
Others	---	---	---	---	---	---	---	---	---	---	---	---
Total	-0.25	-0.13	0.013	0.005	-0.117	-0.249	-0.020	0.067	-0.212	0.213	0.179	-0.119

Example for an fixed income return and risk attribution(4/6)

ANALYSIS 3- INDIVIDUAL BONDS (BEGINNING PERIOD)

Description	Currency	ID	Coupon	Maturity	Price	Return	Selection	% Weight
FRANCE 3.7500%	Euro	F192997*	3.750	25.04.2021	95.10	0.30%	-0.30%	5.10%
DEPFA ACS BANK	Japan	M*064654	1.650	20.12.2016	98.67	-0.03%	0.53%	4.99%
UNITED STATES T	United States	912828GX	2.625	15.07.2017	112.16	-0.44%	0.53%	4.79%
BAYERISCHE LNDB	Japan	M*042210	1.000	20.09.2010	100.74	-0.28%	0.28%	4.50%
UNITED STATES T	United States	912828CA	4.000	15.02.2014	105.27	-0.75%	0.09%	4.08%
SPAIN 4.2000%	Euro	SP12932*	4.200	31.01.2037	93.74	-0.59%	-0.19%	3.80%
KFW INTL FINANC	Japan	M*018014	1.750	23.03.2010	102.34	-0.32%	-0.01%	3.57%
EKSPORTFINANS	Japan	M*018229	1.800	21.06.2010	102.61	-0.31%	0.25%	3.56%
JAPAN HIGHWAY	United States	M*042608	4.625	24.10.2013	104.78	-1.39%	0.51%	3.35%
NETHERLANDS 4.0	Euro	NN00228*	4.000	15.07.2016	100.33	1.04%	0.02%	3.19%
SPAIN 4.4000%	Euro	SP12916*	4.400	31.01.2015	103.12	1.16%	0.13%	2.89%
BK NED GEMEENTE	Japan	767679	0.800	22.09.2008	100.04	-0.39%	0.01%	2.88%
INTERAMER DEV B	Japan	M*017840	1.900	08.07.2009	101.90	-0.40%	-0.03%	2.42%
1.9 % MEDIUM TE	Japan	13508PCQ	1.900	23.03.2009	101.13	-0.25%	0.07%	2.42%
COUNCIL OF EURO	United States	M*027499	6.125	25.01.2011	109.26	-1.37%	0.20%	2.30%
NATIONWIDE LIFE	United Kingdom	M*029619	6.250	28.06.2011	101.99	-1.92%	1.09%	2.21%
USTRIA 4.125%	Euro	LAI0021*	4.125	15.01.2014	102.09	1.06%	-0.02%	1.99%
GEN ELEC CAP CO	Japan	B03225	0.550	14.10.2008	99.58	-0.36%	0.02%	1.97%
GE CAPITAL UK	United Kingdom	M*067192	5.125	03.03.2015	96.05	-2.66%	0.76%	1.95%
EUROPEAN INVT B	Japan	M*079964	1.900	26.01.2026	97.57	0.90%	-0.12%	1.93%
EUR CASH	Euro	EURCASH1	4.107	01.02.2008	100.00	0.33%	0.00%	1.14%

Example for an fixed income return and risk attribution(5/6)

ANALYSIS 4 - TRACKING ERROR ATTRIBUTION (END OF PERIOD)

Currency (Tracking Error, p.a.)	Duration	Term Structure	Sector	Quality	Other Spread	Currency	Covariance	Total
Switzerland	---	---	---	---	---	---	---	---
Euro	0.23%	0.14%	0.01%	0.00%	0.06%	0.00%	-0.18%	0.24%
United Kingdom	0.02%	0.07%	0.03%	0.01%	0.00%	0.00%	-0.05%	0.09%
Denmark	0.02%	0.02%	0.00%	0.00%	0.00%	0.00%	-0.02%	0.02%
Norway	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Sweden	0.01%	0.01%	0.00%	0.00%	0.00%	0.04%	0.01%	0.05%
Czech Republic	---	---	---	---	---	---	---	---
Hungary	---	---	---	---	---	---	---	---
Poland	---	---	---	---	---	---	---	---
United States	0.11%	0.08%	0.15%	0.03%	0.03%	0.02%	-0.14%	0.21%
Canada	0.05%	0.04%	0.00%	0.00%	0.00%	0.04%	-0.06%	0.06%
Japan	0.14%	0.27%	0.06%	0.00%	0.00%	0.02%	-0.14%	0.30%
Australia	0.01%	0.01%	0.00%	0.00%	0.00%	0.02%	0.01%	0.04%
Hong Kong	---	---	---	---	---	---	---	---
New Zealand	---	---	---	---	---	---	---	---
Singapore	---	---	---	---	---	---	---	---
South Korea	---	---	---	---	---	---	---	---
Mexico	---	---	---	---	---	---	---	---
South Africa	---	---	---	---	---	---	---	---
Others	---	---	---	---	---	---	---	---
Covariance	0.06%	0.07%	0.08%	0.01%	0.02%	0.29%	0.38%	0.18%
Total	0.30%	0.33%	0.19%	0.03%	0.06%	0.07%	-0.25%	0.49%

Tracking Error (Annual Std Dev)	0.49%
Portfolio Total Risk (Annual Std Dev)	5.04%
Benchmark Total Risk (Annual Std Dev)	5.28%

VaR Analysis (95%)	Percent	Value
One Month Relative VaR	0.28%	65
One Month VaR	2.87%	672

Example for an fixed income return and risk attribution(6/6)

ANALYSIS 5- INDIVIDUAL BONDS (END OF PERIOD)

Description	% Weight	SEDOL	Coupon	Maturity	Country	Currency	Sector	Quality	Price	Marg Risk	Marg TE
EUROPEAN INVT BK	2.448	M*079964	1.9	26.01.26	YY	JPY	SUPR	Aaa	98.68	0.049	-0.047
KFW	1.559	M*097981	2.6	20.06.37	DE	JPY	AGCY	Aaa	104.39	0.050	-0.045
EUROPEAN INVT BK	1.598	M*069966	1.4	20.06.17	YY	JPY	SUPR	Aaa	100.53	0.045	-0.039
DEPFA ACS BANK	3.538	M*064654	1.65	20.12.16	IE	JPY	ABS	Aaa	98.89	0.046	-0.036
EUR BUND	0.000	FG0806BL	3.75	04.01.17	DE	EUR	TREA	Aaa	116.63	0.031	-0.029
CFWD USD-EUR, 200803	0.105	FX017169			US	EUR	TREA	Aaa	0.66	0.084	-0.027
CFWD GBP-EUR, 200803	0.141	FX017168			GB	EUR	TREA	Aaa	1.31	0.056	-0.025
LIF 10 Yr Gilt	0.000	GT0803LI	8.75	25.08.17	GB	GBP	TREA	Aaa	110.44	0.026	-0.025
SPAIN 4.2000% OBLS	3.793	SP12932*	4.2	31.01.37	ES	EUR	TREA	Tsy	92.86	0.003	-0.023
TSE 10 Yr JGB	0.000	JB0803TS	1.5	20.03.15	JP	JPY	TREA	Aaa	138.47	0.007	-0.021
CBT TNOTE FUTR	0.000	TY0803TY	6	01.03.18	US	USD	TREA	Aaa	118.69	0.028	-0.019
UNITED STATES TREAS	0.314	912810FT	4.5	15.02.36	US	USD	TREA	Tsy	101.27	0.036	-0.018
BAYERISCHE LNDBK	4.512	M*042210	1	20.09.10	DE	JPY	B	Aaa	100.77	0.039	-0.018
EUR BOBL	0.000	FG0803BM	4.25	12.10.12	DE	EUR	TREA	Aaa	111.35	0.018	-0.017
EKSPORTFINANS	3.567	M*018229	1.8	21.06.10	NO	JPY	F	Aa1	102.56	0.039	-0.017
KFW INTL FINANCE	3.572	M*018014	1.75	23.03.10	DE	JPY	AGCY	Aaa	102.28	0.038	-0.016
INTERAMER DEV BK	2.425	M*017840	1.9	08.07.09	YY	JPY	SUPR	Aaa	101.74	0.038	-0.014
CFWD SEK-EUR, 200803	0.003	FX017170			SE	EUR	TREA	Aaa	0.11	0.035	-0.014
1.9 % MEDIUM TERM NO	2.424	13508PCQ	1.9	23.03.09	CA	JPY	TREA	Aaa	101.12	0.038	-0.014
GEN ELEC CAP CORP 0.	1.977	B032Z5	0.55	14.10.08	US	JPY	F	Aaa	99.57	0.037	-0.013
UK 4.2500% GILT DE	1.499	SB16NNR*	4.25	07.12.27	GB	GBP	TREA	Tsy	96.48	-0.003	-0.013
BK NED GEMEENTEN 0.8	2.882	767679	0.8	22.09.08	NL	JPY	B	Aaa	99.98	0.037	-0.013
JPY CASH	0.435	PYCASH1	0.851	01.03.08	JP	JPY	CASH	Aaa	100.00	0.037	-0.012
FRANCE 3.7500% OAT	5.146	F192997*	3.75	25.04.21	FR	EUR	TREA	Tsy	95.10	-0.011	-0.010
CBT T2NTE FUTR	0.000	TU0803TU	6	01.03.10	US	USD	TREA	Aaa	107.66	0.009	-0.005

High
Risk

