Overview of Performance Measurement

Version 2.0
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Agenda (1/4)

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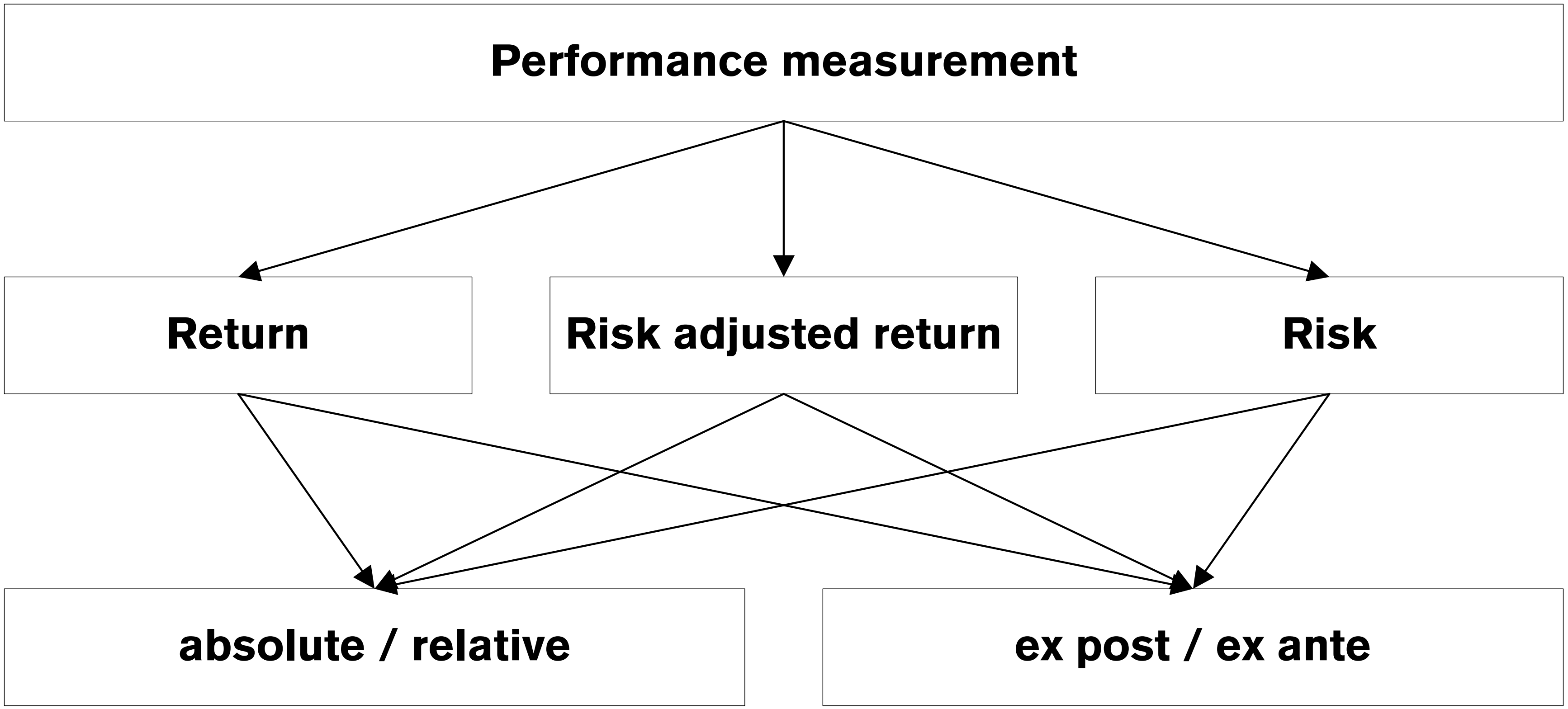
1. Introduction to return calculation

Performance measurement: the big picture



What is a return?

A rate of return is the benefit one received from an investment over a period of time expressed as a percentage. In general the return is the ratio of profit to invested capital.

$$R = \frac{\text{Profit}}{\text{Capital}} = \frac{EMV - BMV}{BMV}$$

R	=	Rate of return
BMV	=	Beginning market value
EMV	=	Ending market value

Important: It seems to be easy but in reality it is not!

Total return versus price return

Normally returns are calculated as total returns. That means that the calculation is based not only on the price changes (capital gains/losses) but also incorporates the earned income, like accrued interest and coupon or dividend payments. In comparison, a price return is only based on the price changes.

$$PR = \frac{P_1 - P_0}{P_0}$$

$$TR = \frac{(P_1 + AI_1) - (P_0 + AI_0) + CD}{(P_0 + AI_0)}$$

PR	=	Price return
TR	=	Total return
P_i	=	Price at point i
AI_i	=	Accrued income at point i
CD	=	Coupon and dividend payments

Net return versus gross return

A net return is the return after the deduction or net of relevant fees and taxes. Due to the wide range of different fees and taxes there are different kinds of net returns. Normally the net return is at least net of transaction costs. On the other hand, a gross return is a return before the deduction or gross of relevant fees and taxes. As for the net return there are different kinds of gross returns.

Important: According to the GIPS Standards gross returns should be presented to prospective clients of asset management companies where the returns are gross of all fees and taxes except for the transaction costs!

Return calculation for specific periods

A return refers always to a specific period. This period can be divided in (several) sub-periods. The return can be calculated over the entire period or by cumulating the returns of the relevant sub-periods. For example: a monthly return can be calculated by cumulating the relevant daily returns.

Important: The cumulation of returns for sub-periods must not lead to the return calculated over the entire period!

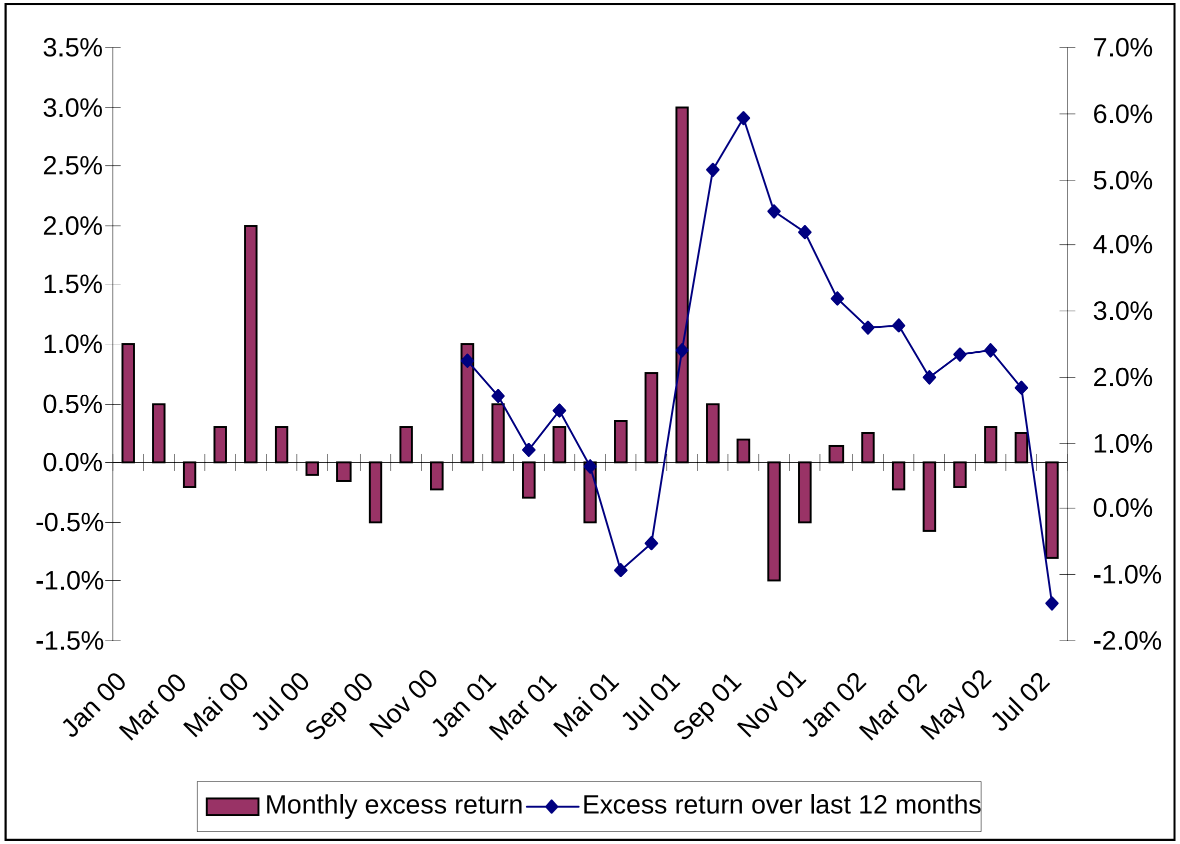
Reporting periods

Returns may refer to different reporting periods:

- fixed period: for example a day, a month, a quarter, a calendar year, etc.
- fixed broken periods: month to date (MTD), quarter to date (QTD), year to date (YTD).
- rolling periods: last day, last month, last 3 months, last year, last 3 years, etc.

Important: Returns are very sensible with respect to the chosen reporting period!

Reporting periods - problems with rolling figures



Important: Rolling periodic returns are very sensitive to the underlying reporting period!

Annualizing returns

An annualized return is the geometric mean return for a 1-year period. In general one can calculate a geometric mean for any period but the 12 month period is common practice. Annualizing returns for less than 12 month is seen as not best practice.

$$\text{Annualized return} = (1 + \text{return over entire period})^{\left(\frac{\text{number of periods per year}}{\text{number of periods}}\right)} - 1$$

$$10\% = (1 + 21\%)^{\left(\frac{1}{2}\right)} - 1$$

Portfolio valuation (1/3)

Returns are mainly based on portfolio valuations done at the beginning and at the end of the calculation period. These portfolio valuations can be done with varying frequency (daily, monthly, quarterly, etc.).

Important: The portfolio valuation is normally based on market/fair values and not on book values - shorter intervals between the portfolio valuation increases heavily the effort for the return calculation!

Portfolio valuation (2/3)

Example for valuation principles (based on the GIPS 2010 exposure draft):

- Fair value is the amount at which an asset could be exchanged in a current arm's length transaction between willing parties in which the parties each acted knowledgeably and prudently.
- Valuation must represent the observable market price of investments or, in the absence of such, must represent management's best estimate of the market price using market-based inputs.
- If investments are valued using subjective, unobservable inputs, and are material to the portfolio, firm must disclose this fact.
- Fair values should be obtained from a qualified independent external third party.

Portfolio valuation (3/3)

Example for valuation hierarchy (based on the GIPS 2010 exposure draft):

- Valuations must be objective, observable, unadjusted market prices in active markets; if not available, then should use.
- Objective, observable quoted market prices for similar investments in active markets; if not available, then should use.
- Quoted prices for identical or similar investments in markets that are not active; if not available, then should use.
- Market based inputs other than quoted prices observable for the investment; if not available, then should use.
- Subjective, unobservable inputs for the investments where markets are not active at the measurement date. Unobservable inputs should only be used to the extent that observable inputs and prices are not available.

Average invested capital

The average invested capital (AIC) is the basis for the return calculation. AIC equals the portfolio value at the beginning of the calculation period if there are no interim (external) cash flows. In the other case, interim (external) cash flows influence the value of the AIC, where cash outflows reduce the AIC and cash inflows increase the AIC.

In practice there are different methods for incorporating cash flows and the returns may vary substantially depending on the chosen methodology.

Important: Cash flows do not refer only to cash payments but also to stock deliveries!

Internal versus external cash flows

The return calculation only considers external and not internal cash flows. External cash flows are cash flows, like cash payments or stock deliveries, which are not done within the portfolio. Coupon or dividend payments are internal cash flows as long as they are paid to an cash account within the portfolio and not to a cash account outside the relevant portfolio.

Important: Cash flows may be treated differently if calculating net and gross returns (for example management or custody fees)!

Impact of external cash flows

Cash flows can be considered as profit relevant or profit irrelevant, which means that the timing effect of the cash flows can be neutralized or not. The time-weighted rate of return (TWR) neutralizes the timing effect of external cash flows and the money-weighted rate of return (MWR) incorporates the timing effect of external cash flows.

Important: TWR should be used to measure the return of a portfolio manager if he has no discretion over external cash flows.

MWR should be used to measure the return of a portfolio manager if he has discretion over external cash flows.

MWR should also be used to measure the return from a client perspective.

Excess versus relative return

Excess return is the arithmetic difference between a portfolio return and the benchmark return.

$$\text{Excess return} = \text{portfolio return} - \text{benchmark return}$$

$$\text{Excess return} = 5.000\% - 4.000\% = 1.000\%$$

Relative return is the geometric difference between a portfolio return and the benchmark return.

$$\text{Relative return} = \frac{1 + \text{portfolio return}}{1 + \text{benchmark return}} - 1$$

$$\text{Relative return} = \frac{1 + 5.000\%}{1 + 4.000\%} - 1 = 0.962\%$$

$$\text{Portfolio return} = (1 + \text{relative return}) \times (1 + \text{benchmark return}) - 1$$

$$5.000\% = (1 + 0.962\%) \times (1 + 4.000\%) - 1$$

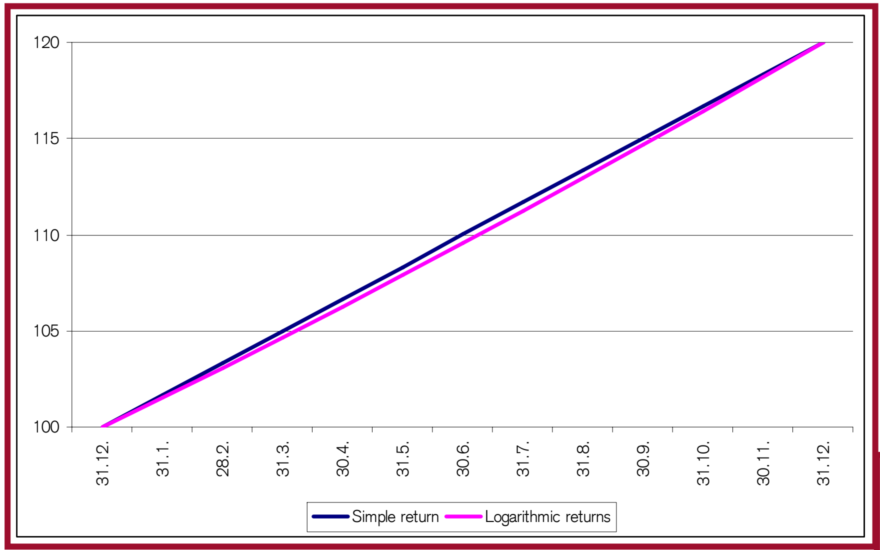
Simple versus logarithmic returns (1/2)

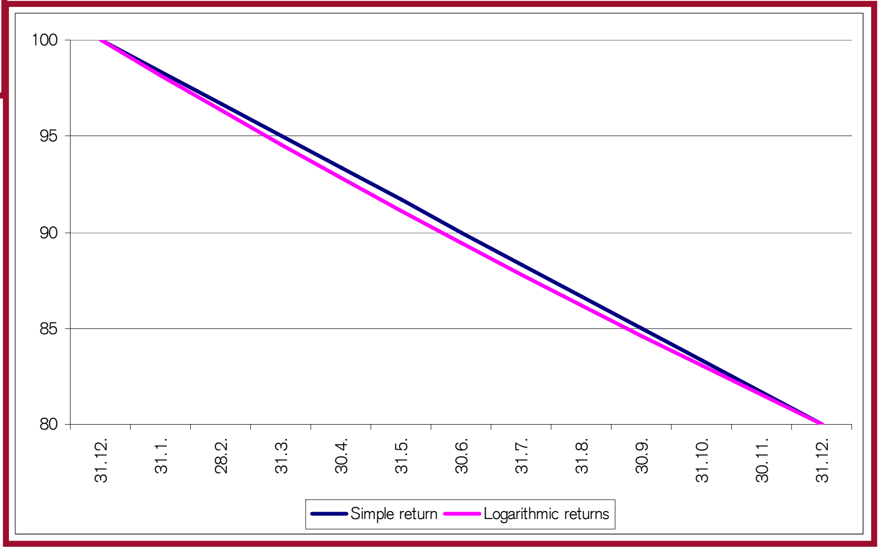
A simple return is a return that assumes a linear interest calculation, meaning that interest is paid at the end of the period or that it is an average interest over the whole period without interest on interest.

A logarithmic return is a return that assumes continuous compounding, meaning that interest is paid for very short sub-periods and that the interest is again reinvested to the same return. Logarithmic returns do not have a basis effect and therefore are often used if calculating risk figures.

$$\text{Log. return} = \ln(1+\text{simple return});\ 18.232\% = \ln(1+20\%);\ -22.314\% = \ln(1-20\%)$$

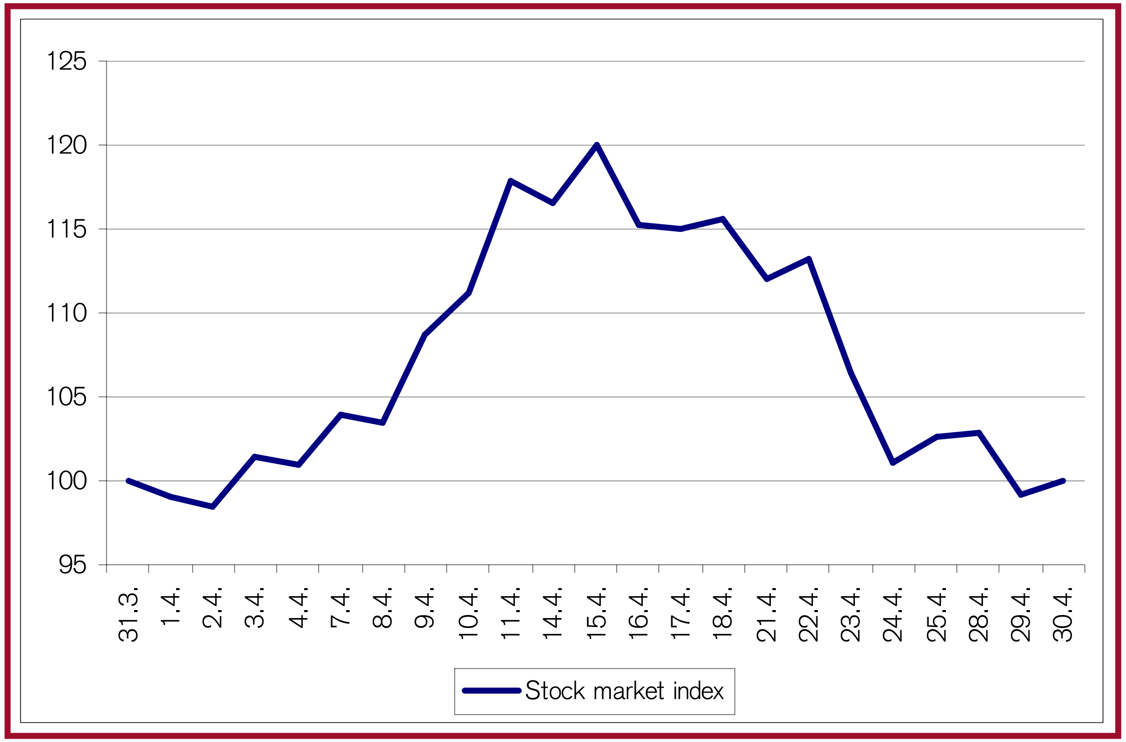
Simple versus logarithmic returns (2/2)





2. Time-weighted versus money-weighted rate of return (TWR versus MWR)

Comparison of TWR versus MWR



What is the return of the stock market index or of an index portfolio manager?

=> TWR => 0%

What is the return of a client who doubled the money invested in the middle of the period?

=> MWR => -11%

Initial comments on TWR

Time-weighted rate of return (TWR) measures the return of a portfolio in a way that the return is insensitive to changes in the money invested:

- TWR measures the return from a portfolio manager’s perspective if he does not have control over the (external) cash flows.
- TWR allows a comparison against a benchmark and across peer groups.
- calculating, decomposing and reporting TWRs is common practice.
- presenting TWRs is one of the key principles of the GIPS Standards.

Decomposition of TWR

The TWR allows a decomposition of the portfolio return reflecting the portfolio manager's main investment decision:

- Benchmark effect => reflects the return contribution based on the client's decision to invest his initial capital into a specific benchmark strategy (corresponds to the benchmark return).
- Management effect => reflects the return contribution based on deviating from the benchmark strategy by asset allocation and stock picking.

Initial comments on MWR

Money-weighted rate of return (MWR) measures the return of a portfolio in a way that the return is sensitive to changes in the money invested:

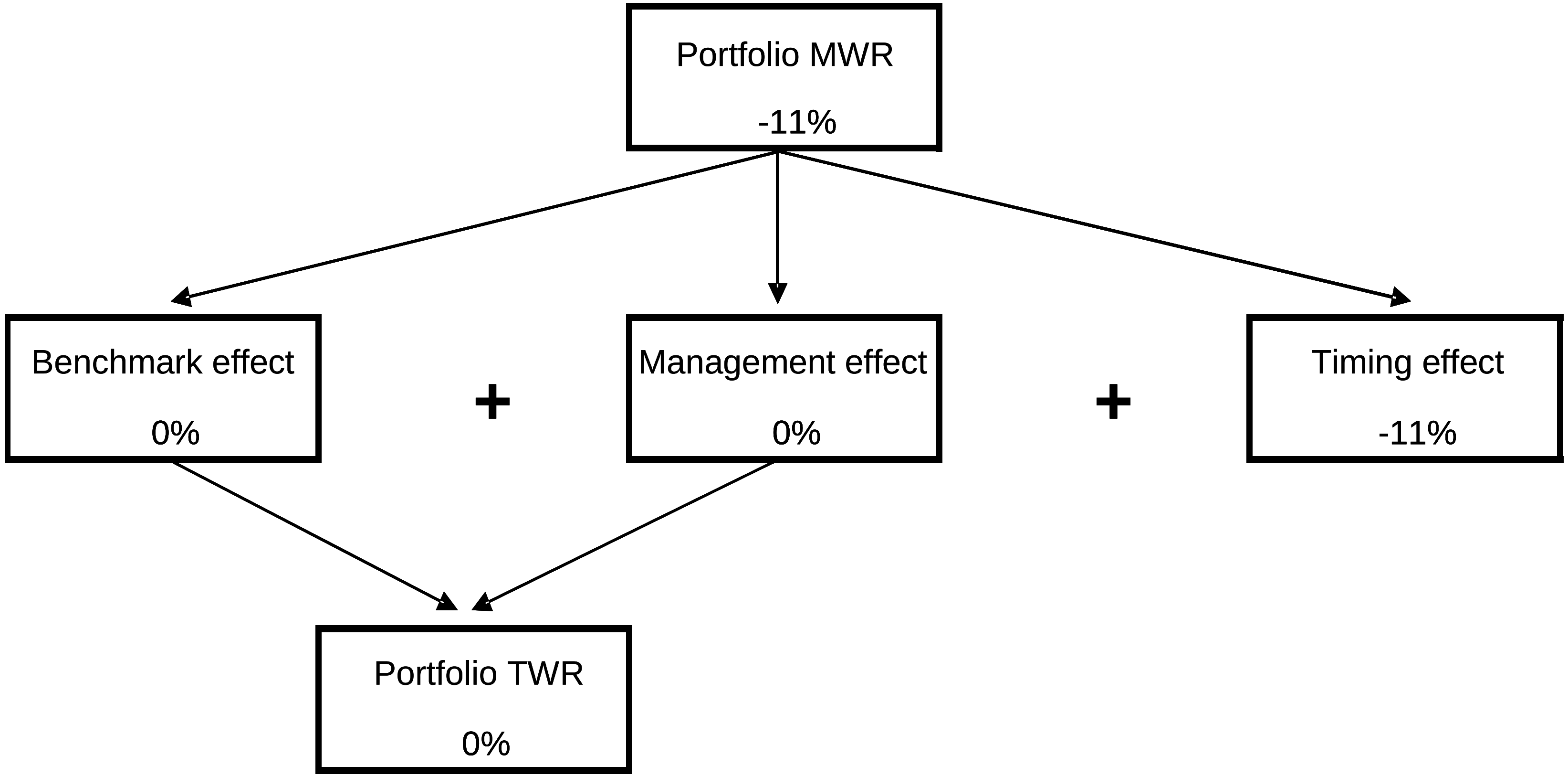
- MWR measures the return from a client's perspective where he does have control over the (external) cash flows.
- MWR does not allow a comparison across peer groups.
- MWR does allow a comparison against a benchmark (adjusted for cash flows).
- MWR is best measured by the internal rate of return (IRR).
- calculating, decomposing and reporting MWRs is not common practice.
- MWRs are not covered by the GIPS Standards.

Decomposition of MWR

The MWR allows a decomposition of the portfolio return reflecting the client's main investment decision:

- Benchmark effect => reflects the return contribution based on the client's decision to invest his initial capital into a specific benchmark strategy (corresponds to the benchmark return).
- Management effect => reflects the return contribution based on deviating from the benchmark strategy by asset allocation and stock picking.
- Timing effect => reflects the return contribution of changing the initial invested capital into the benchmark strategy and into the asset allocation of the portfolio.

Relationship between MWR and TWR



Important: TWR equals the benchmark effect plus management effect!

3. Methods for calculating MWR and TWR

MWR and TWR - Definition

- Money-weighted rate return (MWR) is the average growth rate of all money invested over the entire investment period.
- Time-weighted rate of return (TWR) is the growth rate of a single unit of money invested over the entire investment period.

MWR - Internal rate of return (1/3)

Internal rate of return (IRR) is a MWR and is called the "true" MWR as it is the most precise method for calculating a MWR. The IRR is the return / interest rate that causes the ending value and intermediate cash flows to be discounted to the beginning value.

$$\frac{EMV}{(1+IRR)^T} + \left(\sum_{t=1}^{T-1} \frac{-C_t}{(1+IRR)^t} \right) - BMV = 0$$

IRR	=	Internal rate of return
BMV	=	Beginning market value
EMV	=	Ending market value at T
C_t	=	Cash flows at t

MWR - Internal rate of return (2/3)

Example:

31.3. BMV = 100 → 2.4. C = +10 → 30.4. EMV = 120

$$\frac{120}{(1+IRR)^{\frac{30}{30}}} + \frac{-10}{(1+IRR)^{\frac{2}{30}}} - 100 = 0$$

$$\Rightarrow \quad IRR = 9.149\ \%$$

MWR - Internal rate of return (3/3)

The IRR is also the return / interest rate that causes the beginning value and intermediate cash flows to grow to the ending value and vice versa.

Example:

31.03.	02.04.	30.04.
100.00	10.00	
	→	10.85
→		109.15
		120.00

MWR - Original Dietz Method (1/2)

Original Dietz Method (ODM) is a method to approximate the MWR, where the cash flows are weighted with 0.5 (assuming that cash flows always occur at the middle of the period - normally a month).

$$R = \frac{EMV - BMV - \sum_{i=1}^{n} C_i}{BMV + 0.5 * \sum_{i=1}^{n} C_i}$$

R	=	Rate of return
BMV	=	Beginning market value
EMV	=	Ending market value
C_i	=	Cash flow i

MWR - Original Dietz Method (2/2)

Example:

31.3.
BMV = 100

2.4.
C = +10

30.4.
EMV = 120

$$R = \frac{120 - 100 - 10}{100 + 0.5 * 10} = 9.524\%$$

MWR - Modified Dietz Method (1/2)

Modified Dietz Method (MDM) is a method to approximate the MWR, where the cash flows are daily weighted. The weight of the cash flow depends on the point in time in the relevant period when the cash flow occurred (assumption: cash flows always occur at the end of the day).

$$R = \frac{EMV - BMV - \sum_{i=1}^{n} C_i}{BMV + \sum_{i=1}^{n} W_i * C_i}$$

$$\text{where } W_i = \frac{D - D_i}{D}$$

R	=	Rate of return
BMV	=	Beginning market value
EMV	=	Ending market value
C_i	=	Cash flow i
W_i	=	Weight of cash flow i
D	=	Number of calendar days in period
D_i	=	Day of cash flow i

MWR - Modified Dietz Method (2/2)

Example:

31.3.
BMV = 100

2.4.
C = +10

30.4.
EMV = 120

$$R = \frac{120 - 100 - 10}{100 + \left(\frac{30-2}{30}\right) * 10} = 9.146\%$$

MWR - special case daily Modified Dietz Method

Daily return calculation is based on Modified Dietz Method but assuming that the daily cash flows always occur at the end of the day (this assumption is common practice).

$$R_j = \frac{EMV_j - BMV_j - C_j}{BMV_j}$$

R_j = Rate of return for day j

BMV_j = Beginning market value for day j

EMV_j = Ending market value for day j

C_j = Sum of cash flows for day j

$$R_j = \frac{EMV_j - BMV_j - C_j}{BMV_j + C_j}$$

=> assuming that the daily cash flows always occur at the beginning of the day.

TWR - using sub-period returns (1/2)

TWR is the return that eliminates the timing effect of all external cash flows. To calculate TWRs, one break up the entire investment period in sub-periods, calculate the returns for the sub-periods and then compound them together to derive the TWR for the entire investment period. The boundaries of the sub-periods may be determined by the dates of each cash flow (=> so called flow-based TWR).

The returns for the sub-periods may be approximated by a MWR.

$$R = \prod_{j=1}^{n} (1 + R_j) - 1$$

R = Rate of return for entire investment period

R_j = Rate of return for sub-period j

BMV_j = Beginning market value for sub-period j

EMV_j = Ending market value for sub-period j

C_j = Sum of cash flows for sub-period j

TWR - using sub-period returns (2/2)

Example:

31.3.
BMV = 100

2.4.
C = +10
EMV= 105
BMV = 115

30.4.
EMV = 120

$$R_1 = \frac{115 - 100 - 10}{100} = 5.00\%$$

$$R_2 = \frac{120 - 115}{115} = 4.35\%$$

$$R_{1,2} = \left(1 + \frac{115 - 100 - 10}{100}\right) * \left(1 + \frac{120 - 115}{115}\right) - 1 = (1 + 5.00\%) * (1 + 4.35\%) - 1 = 9.565\%$$

Comparison of TWR and MWR (1/2)

Measures:

- Money-weighted rate return (MWR) is the average growth rate of all money invested over the total investment period.
- Time-weighted rate of return (TWR) is the growth rate of a single unit of money invested over the total investment period.

Effect of external cash flows:

- MWR: Reflects both the timing and amount of money at work over time.
- TWR: Eliminates the effect of both timing and amount of money at work over time.

Comparison of MWR and TWR (2/2)

Statistic represents:

- MWR: The return that reconciles BMV, C and EMV.
- TWR: The return of a single unit of money invested in the portfolio from the beginning to the end of the entire investment period.

Ordering of investment pattern:

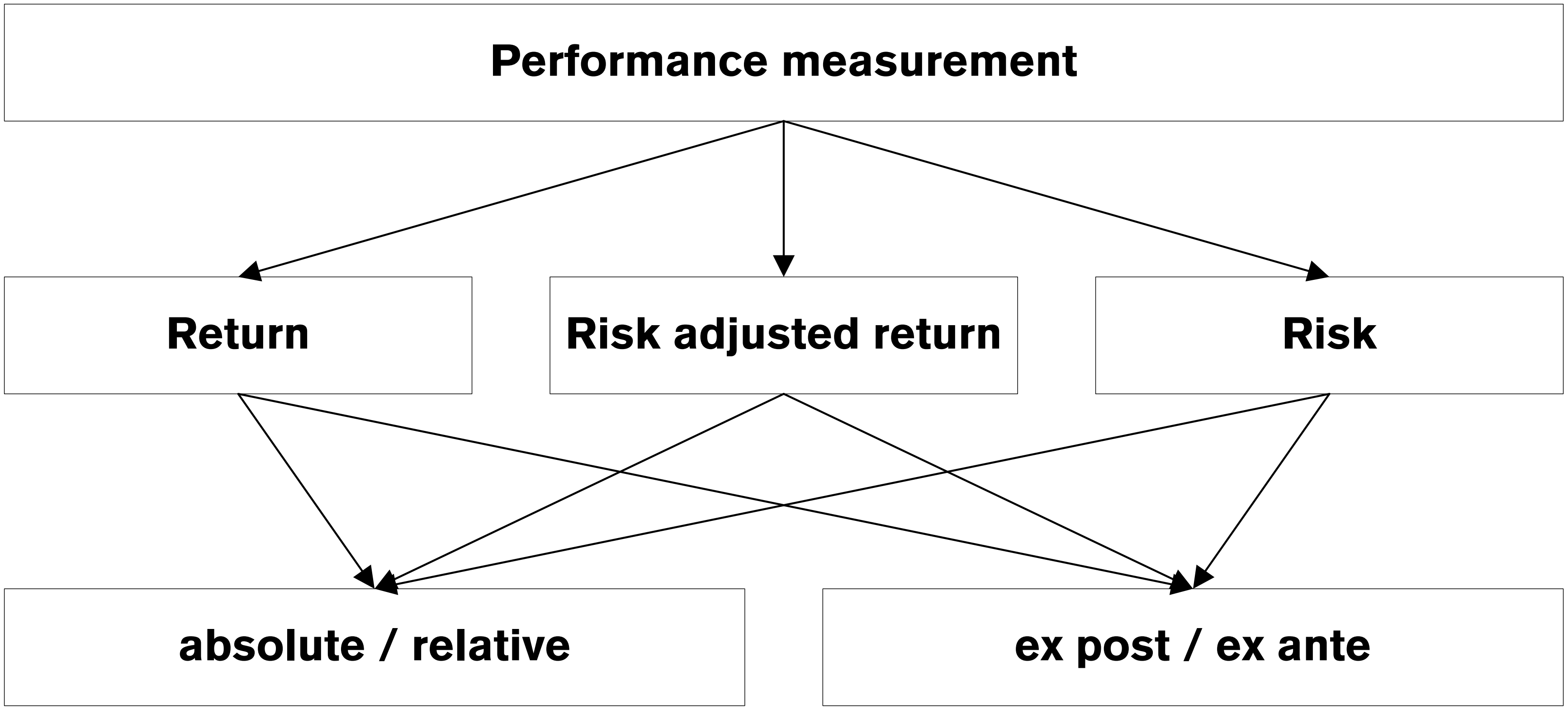
- MWR: Does matter.
- TWR: Does not matter.

Calculation drawbacks:

- MWR: Iteration required for IRR calculation.
- TWR: Portfolio valuation is required before each cash flow.

4. Introduction to risk calculation

Performance measurement: the big picture



What is risk? (1/3)

- There are a lot of definitions for risk. In the following we discuss investment risk as what may be defined as the uncertainty of the expected outcome.
- Considering investment risk we differentiate between:

  a) absolute and relative risk, where the absolute risk focuses on the risk of an investment, a portfolio or a benchmark and the relative risk focuses on the excess risk of a portfolio versus a benchmark.

  b) ex-post and ex-ante risk, where the ex-post risk concentrates on the past and measures the "historical" risk and the ex-ante risk focuses on the expected risk.

What is risk? (2/3)

- In the following we discuss three of the most important figures for measuring investment risk: volatility, tracking error and value at risk.

What is risk?

(3/3)

Reference (RiskNET GmbH, www.risknet.de)

Risk cartoons

Der Blick ins Logbuch ist das eine. RUMMS! ... Risikomanagement das andere.

Risikomanagement ist nicht der Blick in den Rückspiegel

Risiko leitet sich aus dem frühitalienischen 'ris(i)co' ab, die Klippe, die es zu umschiffen gilt.

Leben ist Risiko.
Was wir tun, ist riskant.
Was wir nicht tun, ist es auch.

Wer die Risiken im Blick hat, erkennt die chancenreiche Route.

Volatility (1/3)

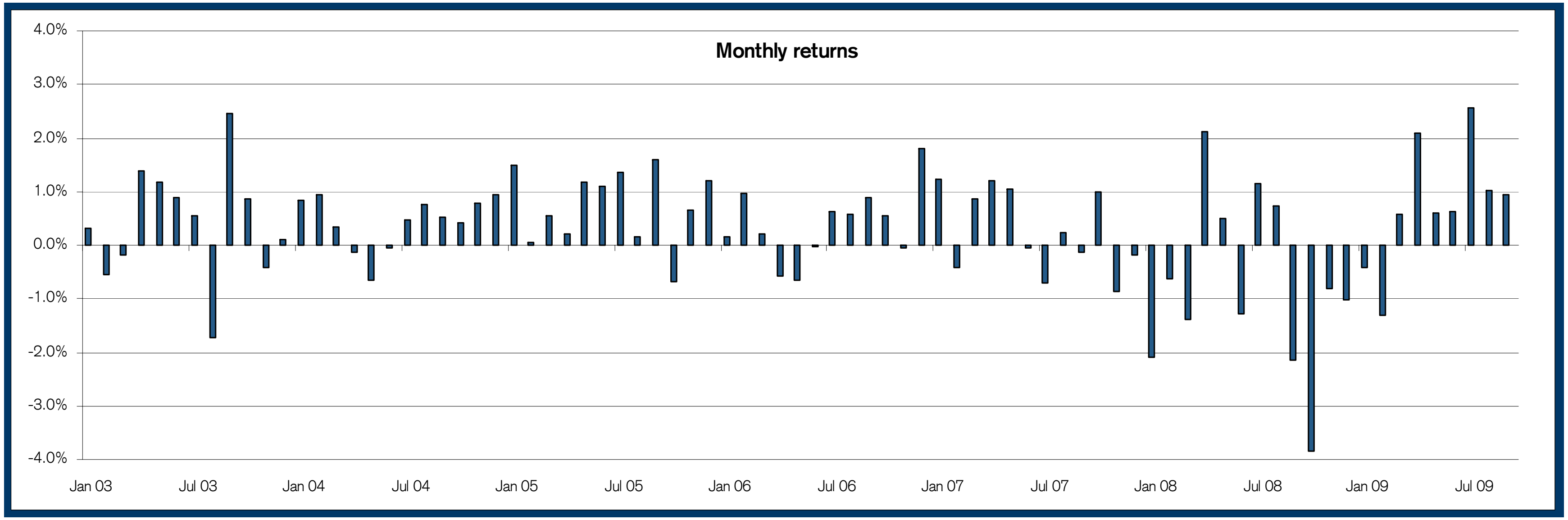
- In the asset management industry the risk of a portfolio is often estimated, measured or expressed using the statistical measure variance (or volatility).
- Variance is easy to interpret, is measured in units of return and is easy to estimate.

$$\sigma^2(R_i) = \frac{1}{n-1} * \sum_{i=1}^{n} \left(R_i - \overline{R}\right)^2 \text{ or}$$

$$\sigma(R_i) = \sqrt{\frac{1}{n-1} * \sum_{i=1}^{n} \left(R_i - \overline{R}\right)^2}$$

$\sigma^2(R_i)$	=	Variance of returns i
$\sigma(R_i)$	=	Volatility of returns i
R_i	=	Historical returns i
$\overline{R}$	=	Average return
n	=	Number of returns

Volatility (2/3)

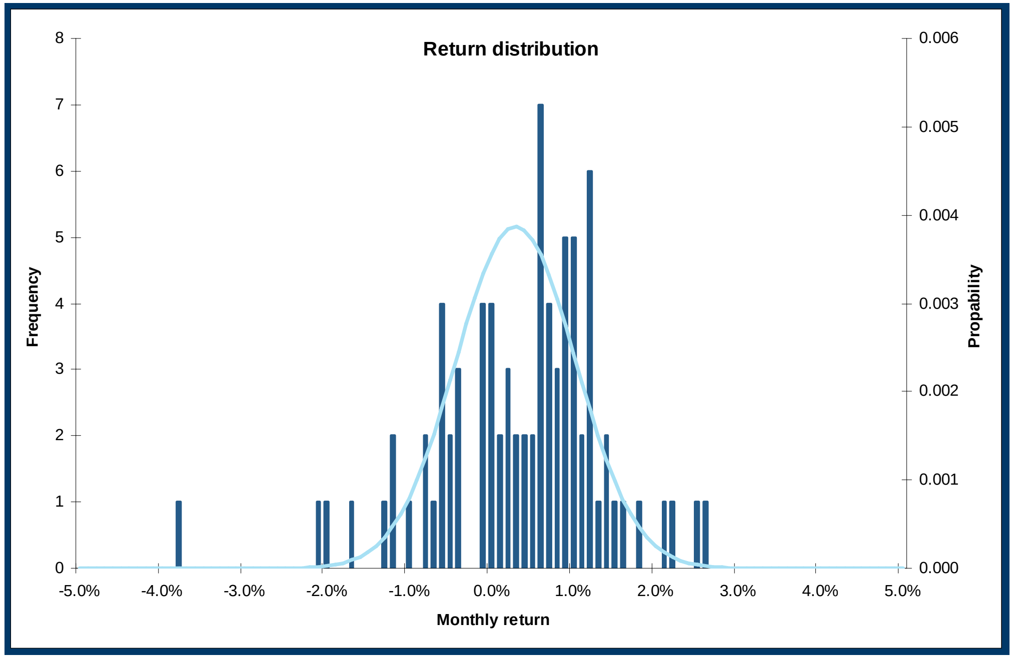


Volatility as a statistical risk figure measures the dispersion of the historical returns around the average of the returns. Interpreted as expected risk, volatility means that the expected returns lay in

a) 67% of all cases between expected return plus/minus one volatility respectively.

b) 95% of all cases between expected return plus/minus two volatilities.

Volatility (3/3)



Annualized return	= 3.6%
Annualized volatility	= 3.7%

Interpretation

=> with a probability of around 67%, over the next 12 months the expected return lays in the area of 3.6% plus/minus 3.7%.

=> with a probability of around 95%, over the next 12 months the expected return lays in the area of 3.6% plus/minus 2 x 3.7%.

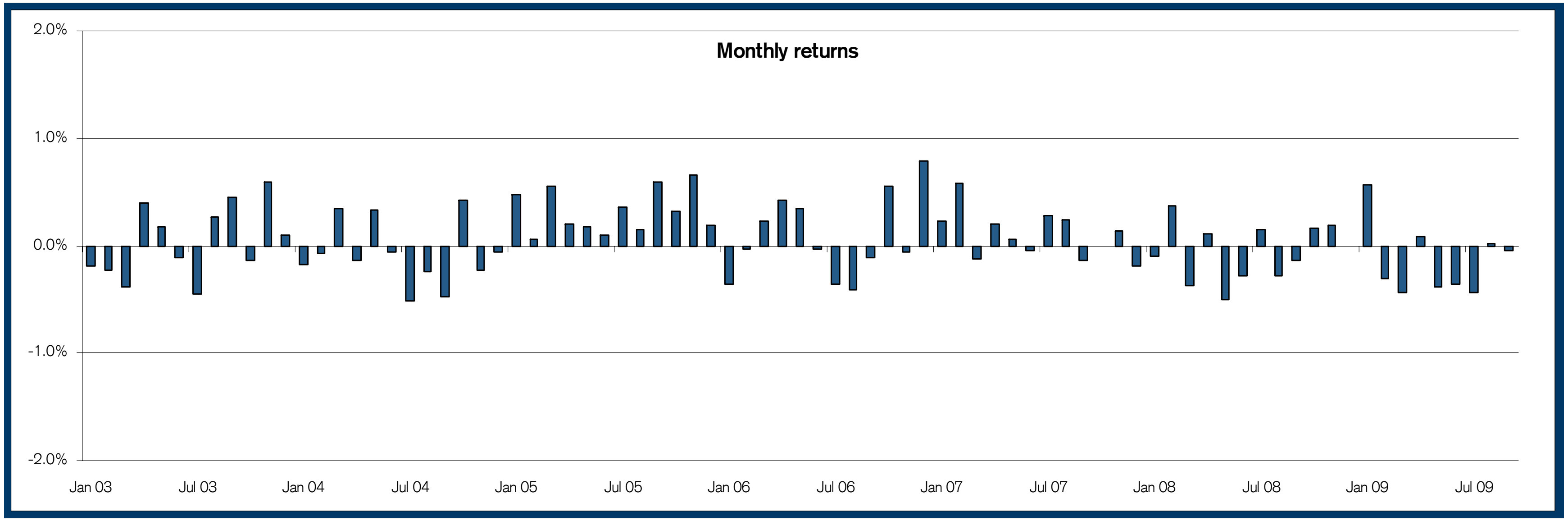
Tracking error (1/2)

- Relative risk is often estimated, measured or expressed using the tracking error, i.e. the variance (volatility) of the excess returns of the portfolio versus the benchmark.

$$ER = R^{P} - R^{BM} \quad \Rightarrow$$
$$\sigma^{2}(ER) = \sigma^{2}\left(R^{P} - R^{BM}\right)$$
$$\sigma^{2}(ER) = \sigma^{2}\left(R^{P}\right) + \sigma^{2}\left(R^{BM}\right) - 2 * Cov\left(R^{P}, R^{BM}\right)$$

ER	=	Excess return
R^{P}	=	Portfolio return
R^{BM}	=	Benchmark return
$Cov\left(R^{P}, R^{BM}\right)$	=	Covariance between R^{P} and R^{BM}

Tracking error (2/2)

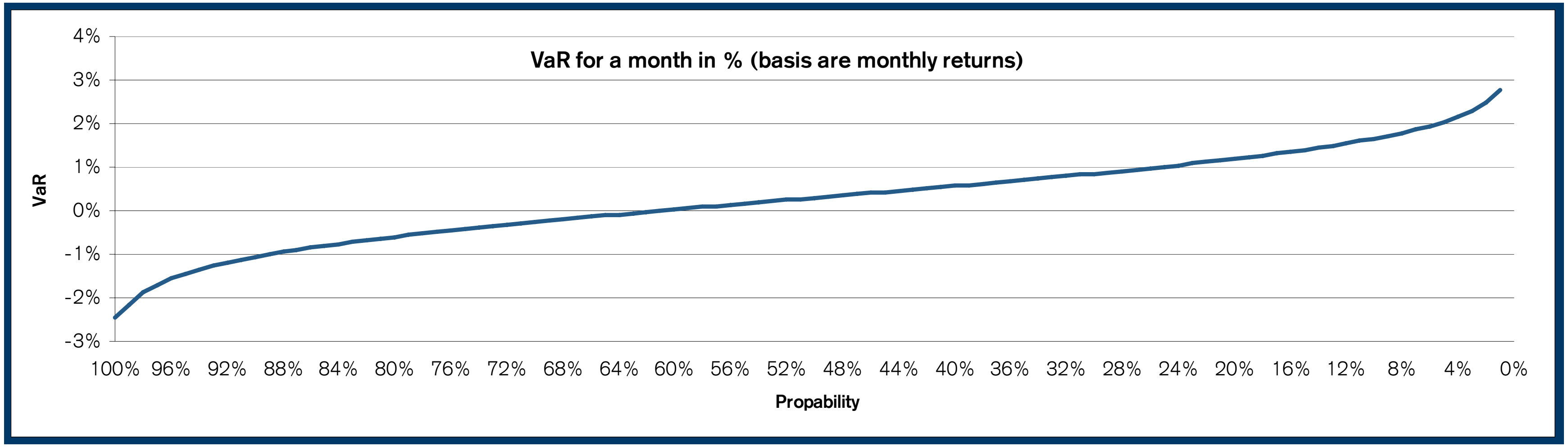


Tracking error as a statistical risk figure measures the dispersion of the historical excess returns around the average of the excess returns. Interpreted as expected risk, tracking error means that the expected excess returns lay in

a) 67% of all cases between expected excess return plus/minus one tracking error respectively.

b) 95% of all cases between expected excess return plus/minus two tracking errors.

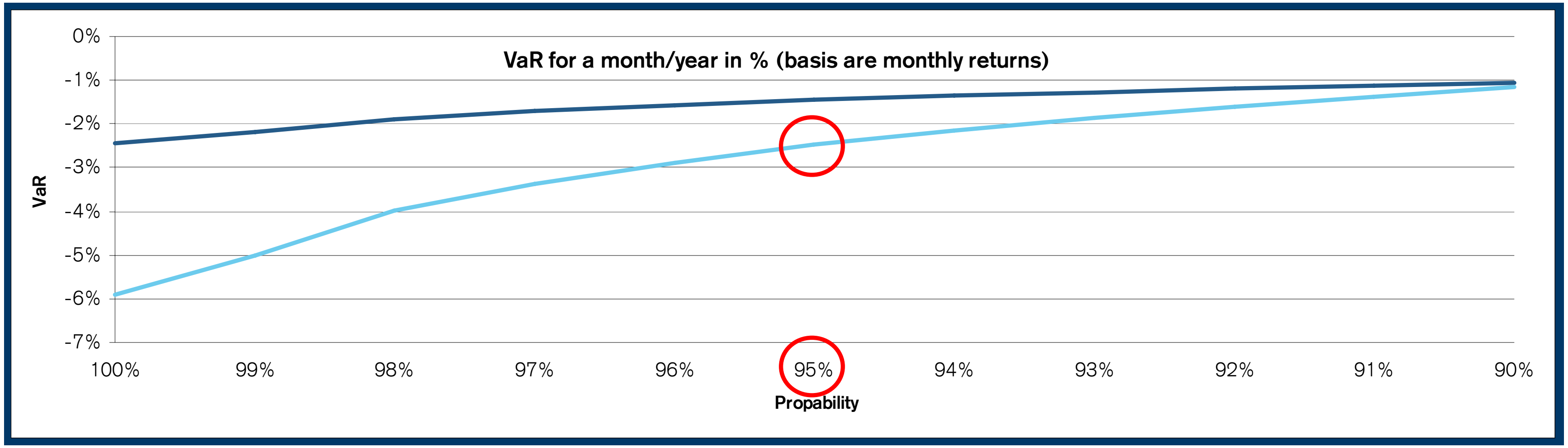
Value at risk (1/2)



Value at Risk (VaR) as a statistical risk figure measures for a given return distribution, for a specific investment period and for a specific probability the absolute or percentage loss.

Interpreted as expected risk, VaR measures the expected absolute or percentage loss which is expected to be not exceeded for a given return distribution, for a specific investment period and for a specific propability.

Value at risk (2/2)

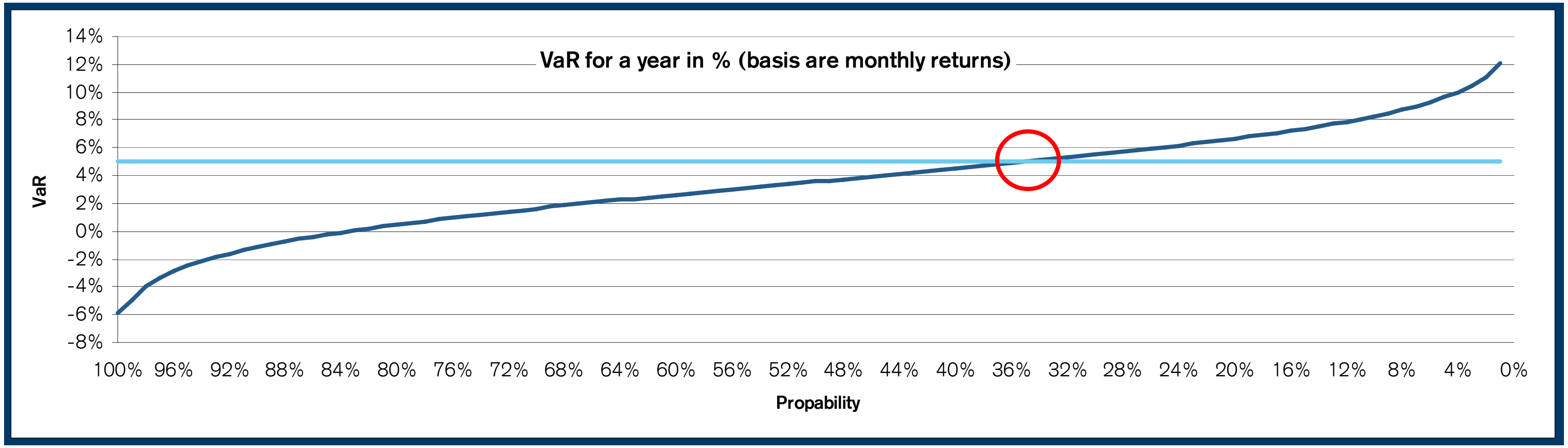


Interpretation

=> with a propability of 95% (99%) is the expected loss (VaR) for a one month period not higher than 1.5% (2.2%) of the total assets.

=> with a propability of 95% (99%) is the expected loss (VaR) for a one year period not higher than 2.5% (5.0%) of the total assets.

Excursus: Value at gain (VaG)

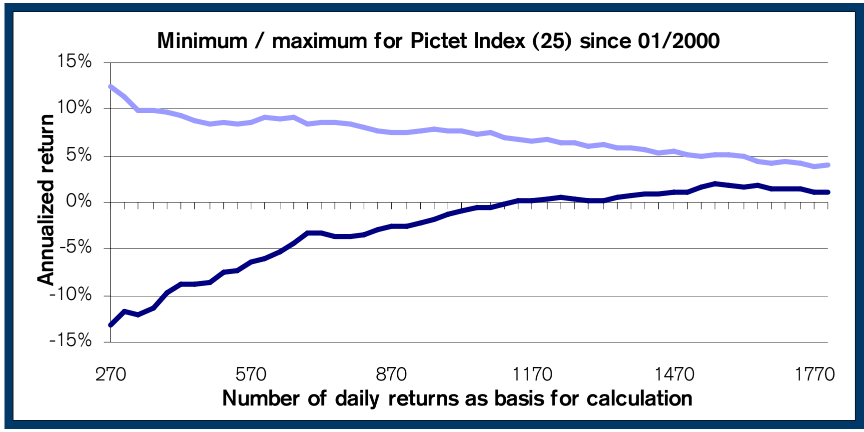


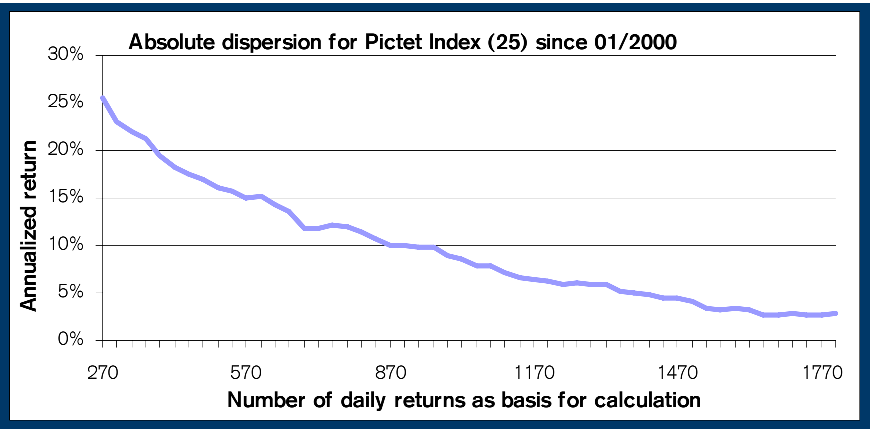
Interpretation

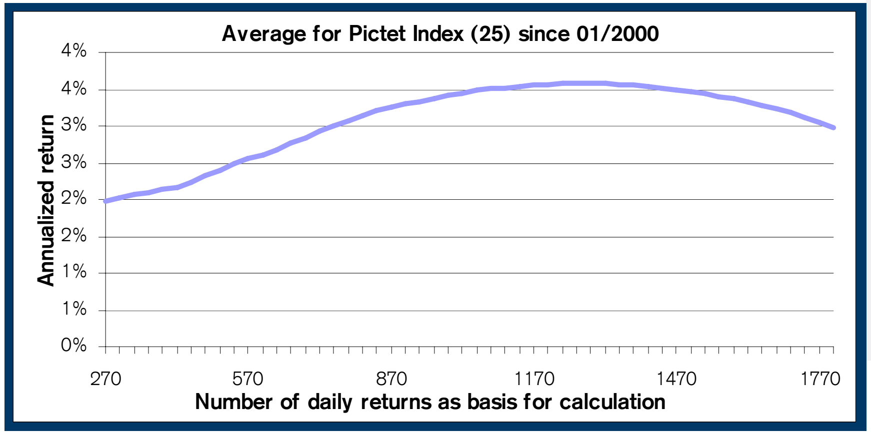
=> with a propability of around 46% is the expected gain (VaG) for a one month period higher than 0.4% of the total assets.

=> with a propability of around 35% is the expected gain (VaG) for a one year period higher than 5.0% of the total assets.

Critical discussion – sensitivity (1/7)



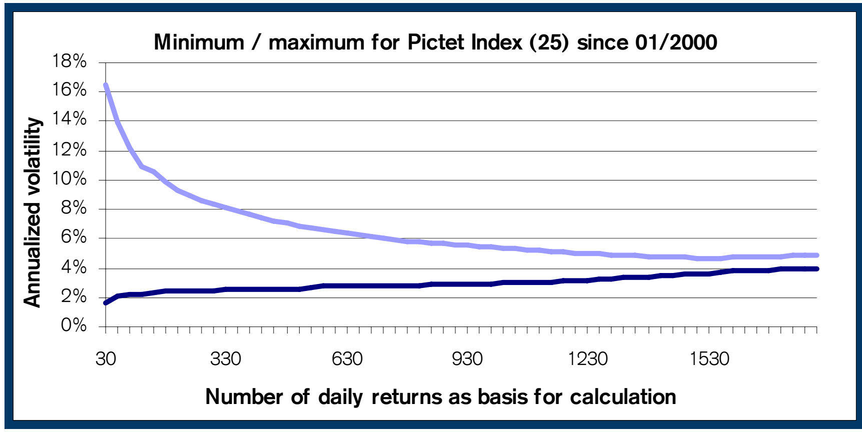


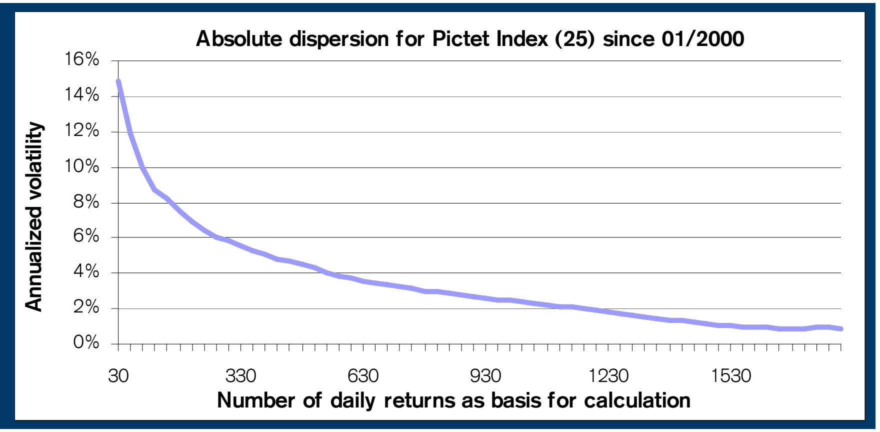


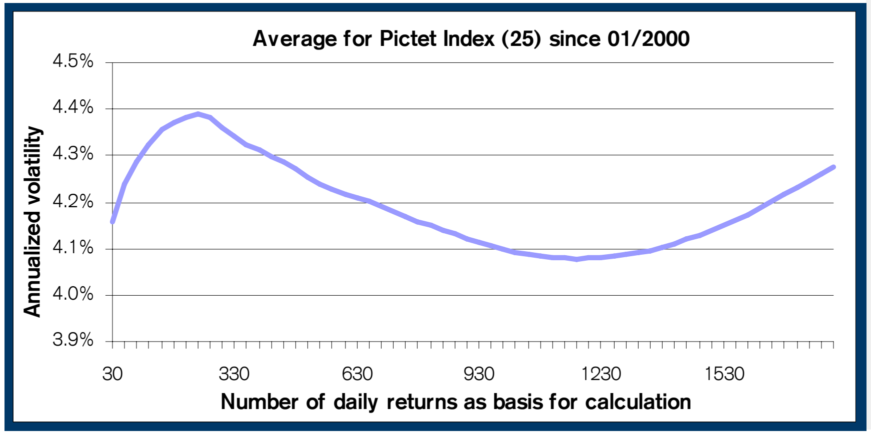
Expected return is very sensitive to the underlying period and length of period (Range for 5-year-returns: 1.6% to 5.8%)

Theory is based on „in the long run ...“

Critical discussion – sensitivity





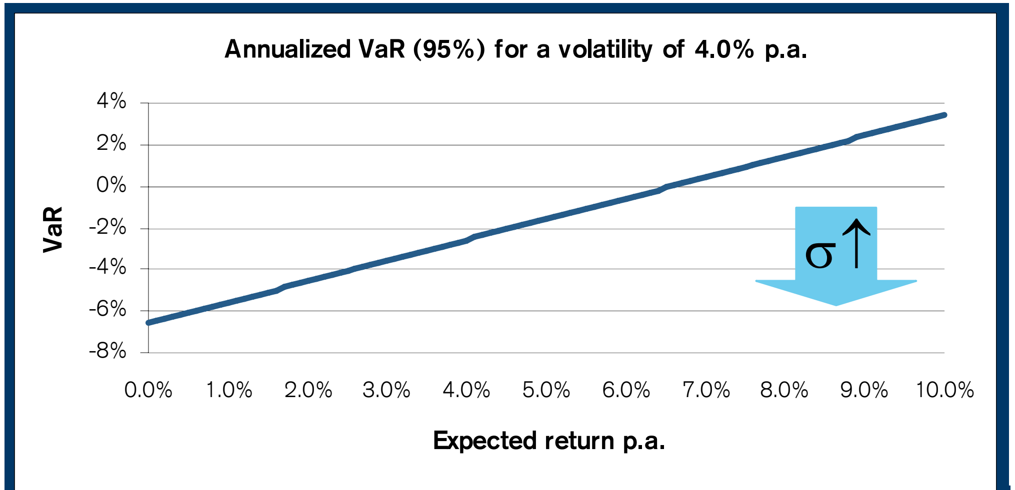


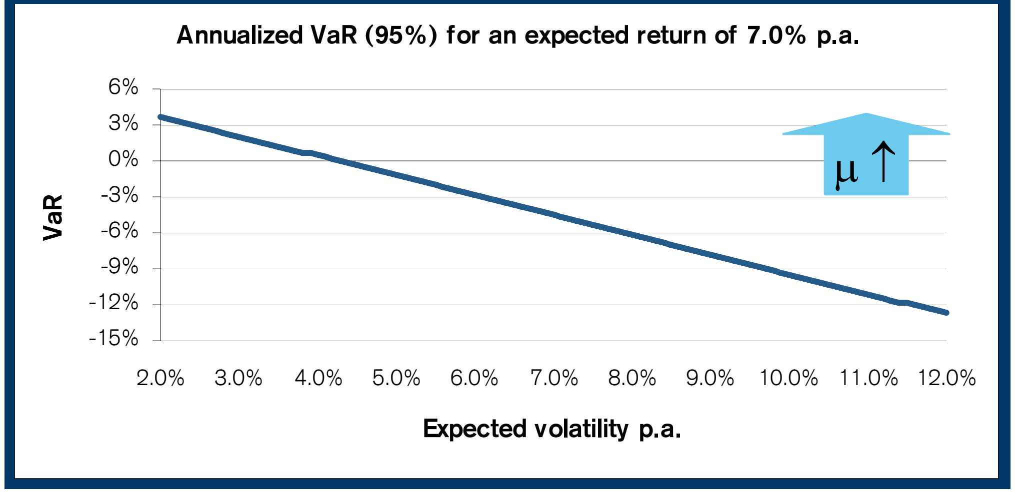
Expected volatility is very sensitive to the underlying period and the length of the period

(Range for 5-year-volatilities:
4.8% to 5.9%)

Theory is based on „in the long run ...“

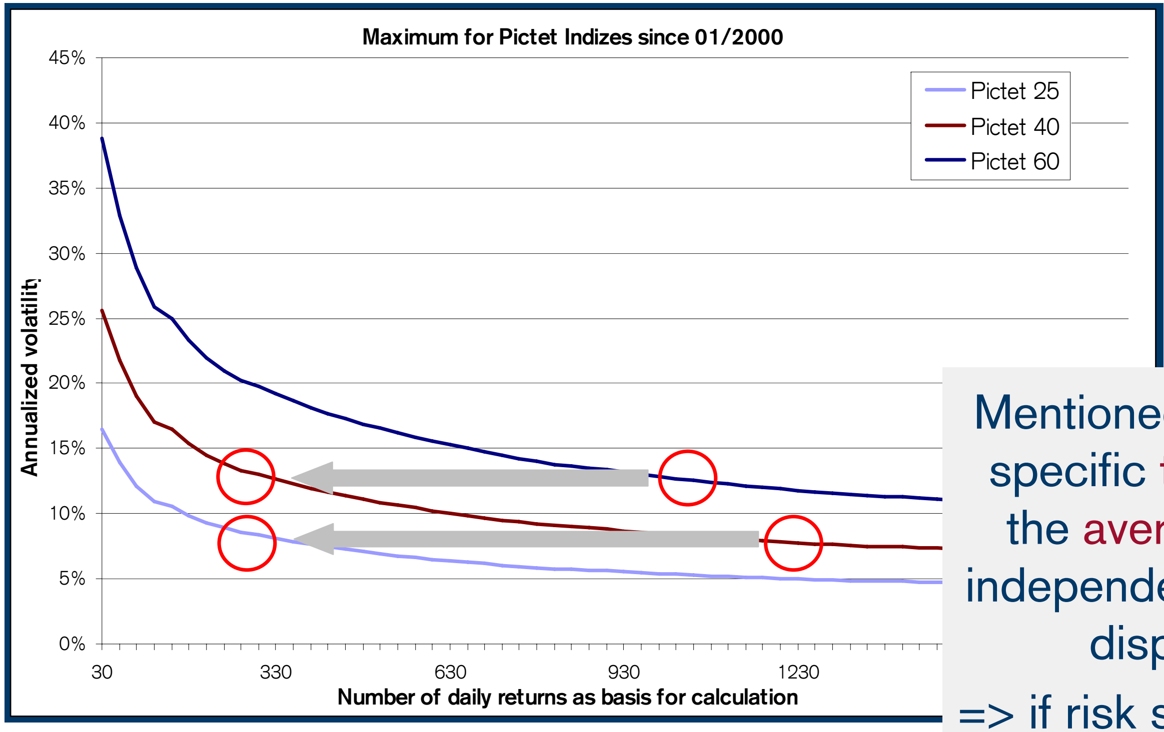
Critical discussion – sensitivity (3/7)





Expected value at risk is very sensitive to the underlying inputs: expected return and expected volatility

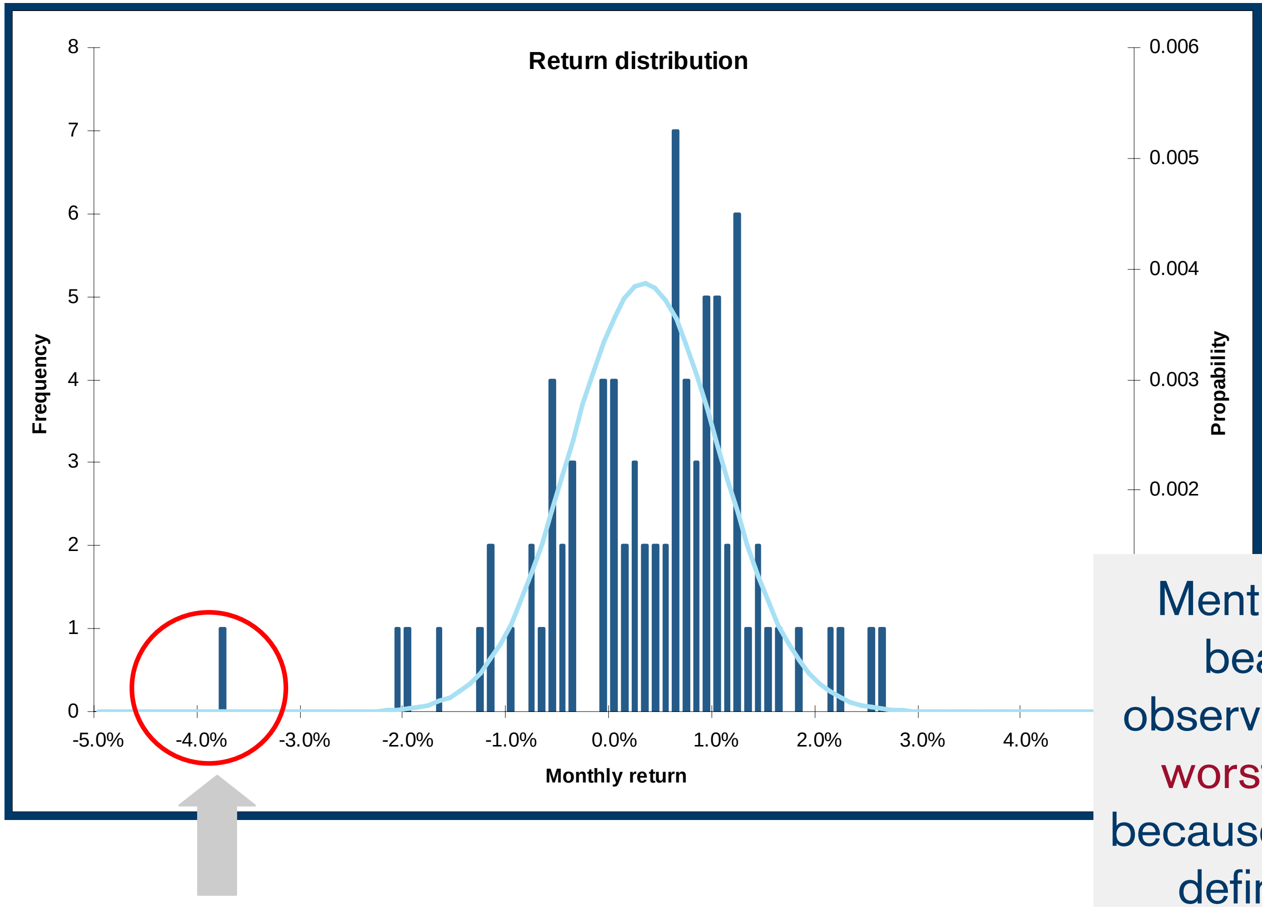
Critical discussion – sensitivity (4/7)



Mentioned risk measures relate to a specific time horizon and describe the average risk for that period – independent from the intertemporary dispersion of the returns

=> if risk should be limited as well for a shorter time period => the relevant time horizon has to be adjusted

Critical discussion – worst case scenario (5/7)



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Mentioned risk measures bear the risk that the observer underestimates the worst case loss potential because the worst case – per definition – occurs very seldom

Critical discussion - summary (6/7)

Advantages

- Easy to interpret.
- Easy to calculate.
- Easy to monitor.
- Common practice in the industry.
- Part of several regulatory requirements.
- Condense the complexity of asset management to a single measure.
- Useful as a long-term risk budget (e.g. within ALM-analysis).

Critical discussion - summary (7/7)

Disadvantages

- Assumption of normal-distributed returns.
- Risk figures are very depended on the underlying inputs (expected volatility, expected return, time horizon, etc.) – inputs are an implicit estimation!
- Established for „normal“ times at the financial markets and not for stress scenarios – therefore underestimation of the worst case risk.

What other risk measures?

- Sharpe ratio
- Systematic risk
- Sortino ratio
- M^2
- Treynor ratio
- Jensen alpha
- Residual risk
- Information ratio
- Downside risk
- Correlation
- R^2
- Omega
- Beta
- and much more